The case of more than one choice variable

;

Examples

1)

2)

3)

Second order condition

The second order necessary condition:

for max z: for arbitrary values of dx and dy, not both zero

for min z:

<0 iff <0;

>0 iff >0; ;

Conditions for relative extremum: z=f(x,y)

|  |  |  |
| --- | --- | --- |
|  | max | min |
| First-order necessary condition |  |  |
| Second-order sufficient condition | , | , |

Example

=> y=-x

=>

doesn't satisfy second order conditions saddle point ( the sign of will in that case be indefinite positive for some values of dx and dy but negative)

20>4

Example 2

=>

=>

- max point

Positive and negative definitiveness (Determinate test for sign positiveness)

positive definite iff >0;

negative definite iff <0

H= - Hessian determinant

Objective functions with more than two variables

=more than one choice variable

z is max iff ( positive definite)

min ( negative definite)

- Hessian Determinant

Example

+2

Homogenous linear equation system, single solution

Example 2

Find the extreme values

|

n-variable case

Determinantal Test for Relative Extremum:

|  |  |  |
| --- | --- | --- |
|  | Max | Min |
| First-order necessary condition |  |  |
| Second-order sufficient condition | ; …; |  |

Example

1. Find the extreme values of each of the following functions, and determine whether they are maxima or minima

Ex.

Positive Definite