Chapter 9. Optimization

Optimum values and Extreme values

$$y=f\left(x\right)$$

First Derivative Test

if the first derivative of a function f(x) if $x=x\_{0}$ is $f^{'}\left(x\_{0}\right)=0$, then the value of the function at $x\_{0}$, $f(x\_{0})$ will be

a) relative max if f'(x) from + to -

b) relative min if f'(x) from - to +

c) relative a relative max nor relative min if f'(x) has the same sign on lotn the immediate left and the inmteringt of point $x\_{0}$.

$$y=f\left(x\right)=x^{3}-12x^{2}+36x+8$$

$$f^{'}\left(x\right)=3x^{2}-24x+36=0$$

$$f^{'}\left(x\right)=x^{2}-8x+12=0$$

$$D=64-48=16$$

$$x\_{1,2}=\frac{8\pm 4}{2}=6,2$$

Second and Higher Derivatives

$$f^{''}\left(x\right) or \frac{d^{2}y}{dx^{2}}=\frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$f^{'''}\left(x\right),f^{IV}\left(x\right),…,f^{n}\left(x\right)$$

$$\frac{d^{3}y}{dx^{3}},\frac{d^{4}y}{dx^{4}},…, \frac{d^{n}y}{dx^{n}};$$

find the first through the fifth derivatives of the function

$$y=f\left(x\right)=4x^{4}-x^{3}+17x^{2}+3x-1$$

$$y=g\left(x\right)=\frac{1}{1+x}$$

Interpretation of the second derivative

$$\left.\begin{array}{c}f^{'}\left(x\_{0}\right)>0\\f^{'}\left(x\_{0}\right)<0\end{array}\right\}$$

means that the value of the function tends to increase, decrease.

$\left.\begin{array}{c}f^{''}\left(x\_{0}\right)>0\\f^{''}\left(x\_{0}\right)<0\end{array}\right\}$

means that the slope of the curve tends to increase, decrease

Second-Derivative Test for relative extremum if the value of the first derivative of a function f at $x=x\_{0}$is $f^{'}\left(x\_{0}\right)=0$, then the value of the function at $x\_{0}$, $f(x\_{0})$ will be

a) A relative max if $f^{''}\left(x\_{0}\right)<0$

b) A relative min if $f^{''}\left(x\_{0}\right)>0$

This test is more convenient