**Guideline for the Problem Set for Lecture 4a: Outside Financing Capacity**

**Problem 1:** Assume a project has a probability of success of *pH=0.8* if the borrower behaves and *pL=0* if the borrower misbehaves. The size of the investment is *I = $1,000,000*. If successful, the project earns $1,300,000. Selling off the equipment after it has been purchased would earn the borrower $500,000. The risk-free interest rate is zero.

a) How much income can the borrower maximally pledge?

pledge = pH (R – B/p) = 0.8 (1,300,000 – 500,000/0.8) =0.8 \* 675,000 = 540,000

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b) State the zero-profit condition for the lender! How much could the lender maximally finance and still meet the zero profit condition if the borrower provides zero equity?

The zero profit condition states: pH Rl = I

As Rb ≥ 675,000 and Rl ≤ 1,300,000 – Rb ≤ 625,000

Maximal investment by full external finance Ifull:

pH Rl = (I – A) 🡺 for A = 0: Ifull = 500,000

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c) How much of her own wealth does the borrower need to invest?

(I – A) = pledge = pH (R – B/p) 🡺 for I = 1,000,000 = 0: Amin = 460,000

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d) Verify that the borrower actually wants to behave and that the investor fulfils its participation constraint!

Rb = R – Rl = 625,000 fulfils Rb ≥ B/p = 500,000/0.8, so borrower wants to behave

Rl ≥ (I – A)/pH or 0.8\*675,000≥ 1,000,000 – 460,000 = 540,000, so participation constraint of the lender is fulfilled.

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e) How small would the scrap value need to be to allow for a contract where no own wealth of the borrower is invested?

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For I = pledge = pH (R – B/p) no own money needs to be invested. The critical private benefit B\* is:

So I = 0.8 (R – B\*/p) or 1,000,000 = 0.8 (1,300,000 – B\*/0.8) 🡺 B\*/0.8 = 40,000 or B\* = 50,000.