**Lecture 8**

**Differentials and Derivatives**

(by definition)

where ,

or

this equation describes the change in y ( that results from a specific not necessarily small change in x ( from any starting value of x in the domain of the function y.

y=f(x)

y B

D

A C

x

x0

Y y=f(x)

dy

A dx

x

AD – a tangent line

DB - the discrepancy of error of approximation as decreases B →A making a better approximation of .

- slope of tangent AD=f'(x)

**Differentials and Point Elasticity**

- demand function

,

-ratio of the marginal function to the average function of the demand function.

For other function y=f(x) the point elasticity of y with respect to x.

Example 1.

= -1

>1 =>25<p<50

>1 =>0<p<25

Example 2.

Find the point elasticity of supply , whether the supply is elastic at P=2

P=2, => supply is elastic

y

A

QA QM

0 X0 B x **Inelastic**

y y=f(x)

A

QA QM x  **Elastic**

**0**

**y**

C

QA QM

0 X0 x QA=QM

**Unitary Elastic**

If AB is steeper the OA, the function is elastic at point A; in the opposite case it is inelastic at A.

Find the differentials for given functions:

1) a)

b)

c)

2) =>

3) C=a+bY

=b

for ; a>0; 0<a<1; b>0 => Yb<a+bY=>

4)

Q- quantity demanded, P- price, Y -income,

given P=20, Y=5000 find the:

a) price elasticity of demand

b) income elasticity of demand

price inelastic.

Income elasticity is positive, so the product is normal good.

**Total Differentials**

The concept of differentials can easily be extended to a function of 2 or more independent variables. S=s(Y,i) - saving function - to this function is assumed to be continuous and to possess continuous ( partial) derivatives.

-Total differential of the saving function.

The process of finding such a total differential is called a total differentiation.

It is possible, that Y may change while i remains constant. In that case, di=0,

More general case

U=U( - if this utility function , marginal utility,

These are partial elasticity.

Example 1. Find the tottal differential for the following utility functions where a,b>0

a)

b)

c)

**Total derivatives**

y=f(x,w) where x=g(w)

y=f[g(w),w]

- total derivative, - partial derivative

Example 1. Find the total derivative dy/dw

As a check, we may substitute the function g into the function f, to get

A variation

where

Another variation

where

Example. , let be the production function, where, aside from the two inputs K and L, there is a third argument t, denoting time. This is a dynamic production function,

K=K(t),

L=L(t)