CHAPTER 8

Economic Growth I: Capital Accumulation and Population Growth

> *The question of growth is nothing new but a new disguise for an age-old issue, one which has always intrigued and preoccupied economics: the present versus the future.*
>
> —*James Tobin*

If you have ever spoken with your grandparents about what their lives were like when they were young, most likely you learned an important lesson about economic growth: material standards of living have improved substantially over time for most families in most countries. This advance comes from rising incomes, which have allowed people to consume greater quantities of goods and services.

To measure economic growth, economists use data on gross domestic product, which measures the total income of everyone in the economy. The real GDP of the United States today is more than five times its 1950 level, and real GDP per person is more than three times its 1950 level. In any given year, we also observe large differences in the standard of living among countries. Table 8-1 shows the 2010 income per person in the world's 14 most populous countries. The United States tops the list with an income of $47,140 per person. Bangladesh has an income per person of only $640—less than 2 percent of the figure for the United States.

Our goal in this part of the book is to understand what causes these differences in income over time and across countries. In Chapter 3 we identified the factors of production—capital and labor—and the production technology as the sources of the economy's output and, thus, of its total income. Differences in income, then, must come from differences in capital, labor, and technology.

Our primary task in this chapter and the next is to develop a theory of economic growth called the **Solow growth model**. Our analysis in Chapter 3 enabled us to describe how the economy produces and uses its output at one point in time. The analysis was static—a snapshot of the economy. To explain why our national income grows, and why some economies grow faster than others, we must broaden our analysis so that it describes changes in the economy over time. By developing such a model, we make our analysis

TABLE 8-1

International Differences in the Standard of Living

Country	Income per person (2010)	Country	Income per person (2010)
United States	$47,140	Indonesia	2,580
Germany	43,330	Philippines	2,050
Japan	42,150	India	1,340
Russia	9,910	Nigeria	1,180
Brazil	9,390	Vietnam	1,100
Mexico	9,330	Pakistan	1,050
China	4,260	Bangladesh	640

Source: The World Bank.

dynamic—more like a movie than a photograph. The Solow growth model shows how saving, population growth, and technological progress affect the level of an economy's output and its growth over time. In this chapter we analyze the roles of saving and population growth. In the next chapter we introduce technological progress.[1]

8-1 The Accumulation of Capital

The Solow growth model is designed to show how growth in the capital stock, growth in the labor force, and advances in technology interact in an economy as well as how they affect a nation's total output of goods and services. We will build this model in a series of steps. Our first step is to examine how the supply and demand for goods determine the accumulation of capital. In this first step, we assume that the labor force and technology are fixed. We then relax these assumptions by introducing changes in the labor force later in this chapter and by introducing changes in technology in the next.

The Supply and Demand for Goods

The supply and demand for goods played a central role in our static model of the closed economy in Chapter 3. The same is true for the Solow model. By considering the supply and demand for goods, we can see what determines how much output is produced at any given time and how this output is allocated among alternative uses.

[1]The Solow growth model is named after economist Robert Solow and was developed in the 1950s and 1960s. In 1987 Solow won the Nobel Prize in economics for his work on economic growth. The model was introduced in Robert M. Solow, "A Contribution to the Theory of Economic Growth," *Quarterly Journal of Economics* (February 1956): 65–94.

The Supply of Goods and the Production Function The supply of goods in the Solow model is based on the production function, which states that output depends on the capital stock and the labor force:

$$Y = F(K, L).$$

The Solow growth model assumes that the production function has constant returns to scale. This assumption is often considered realistic, and, as we will see shortly, it helps simplify the analysis. Recall that a production function has constant returns to scale if

$$zY = F(zK, zL)$$

for any positive number z. That is, if both capital and labor are multiplied by z, the amount of output is also multiplied by z.

Production functions with constant returns to scale allow us to analyze all quantities in the economy relative to the size of the labor force. To see that this is true, set $z = 1/L$ in the preceding equation to obtain

$$Y/L = F(K/L, 1).$$

This equation shows that the amount of output per worker Y/L is a function of the amount of capital per worker K/L. (The number 1 is constant and thus can be ignored.) The assumption of constant returns to scale implies that the size of the economy—as measured by the number of workers—does not affect the relationship between output per worker and capital per worker.

Because the size of the economy does not matter, it will prove convenient to denote all quantities in per-worker terms. We designate quantities per worker with lowercase letters, so $y = Y/L$ is output per worker, and $k = K/L$ is capital per worker. We can then write the production function as

$$y = f(k),$$

where we define $f(k) = F(k, 1)$. Figure 8-1 illustrates this production function.

The slope of this production function shows how much extra output a worker produces when given an extra unit of capital. This amount is the marginal product of capital MPK. Mathematically, we write

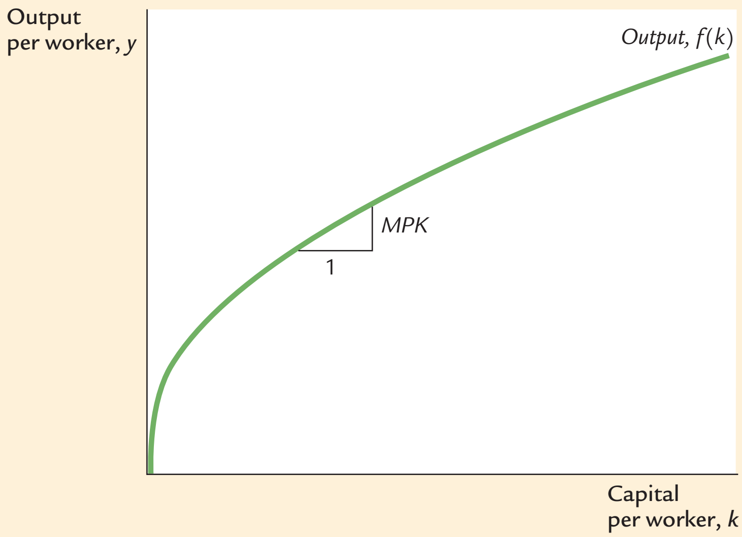
$$MPK = f(k + 1) - f(k).$$

Note that in Figure 8-1, as the amount of capital increases, the production function becomes flatter, indicating that the production function exhibits diminishing marginal product of capital. When k is low, the average worker has only a little capital to work with, so an extra unit of capital is very useful and produces a lot of additional output. When k is high, the average worker has a lot of capital already, so an extra unit increases production only slightly.

The Demand for Goods and the Consumption Function The demand for goods in the Solow model comes from consumption and investment. In other words, output per worker y is divided between consumption per worker c and investment per worker i:

$$y = c + i.$$

FIGURE 8-1



The Production Function The production function shows how the amount of capital per worker k determines the amount of output per worker $y = f(k)$. The slope of the production function is the marginal product of capital: if k increases by 1 unit, y increases by *MPK* units. The production function becomes flatter as k increases, indicating diminishing marginal product of capital.

This equation is the per-worker version of the national income accounts identity for an economy. Notice that it omits government purchases (which for present purposes we can ignore) and net exports (because we are assuming a closed economy).

The Solow model assumes that each year people save a fraction s of their income and consume a fraction $(1 - s)$. We can express this idea with the following consumption function:

$$c = (1 - s)y,$$

where s, the saving rate, is a number between zero and one. Keep in mind that various government policies can potentially influence a nation's saving rate, so one of our goals is to find what saving rate is desirable. For now, however, we just take the saving rate s as given.

To see what this consumption function implies for investment, substitute $(1 - s)y$ for c in the national income accounts identity:

$$y = (1 - s)y + i.$$

Rearrange the terms to obtain

$$i = sy.$$

This equation shows that investment equals saving, as we first saw in Chapter 3. Thus, the rate of saving s is also the fraction of output devoted to investment.

We have now introduced the two main ingredients of the Solow model—the production function and the consumption function—which describe the economy at any moment in time. For any given capital stock k, the production function $y = f(k)$ determines how much output the economy produces, and the saving rate s determines the allocation of that output between consumption and investment.

Growth in the Capital Stock and the Steady State

At any moment, the capital stock is a key determinant of the economy's output, but the capital stock can change over time, and those changes can lead to economic growth. In particular, two forces influence the capital stock: investment and depreciation. *Investment* is expenditure on new plant and equipment, and it causes the capital stock to rise. *Depreciation* is the wearing out of old capital, and it causes the capital stock to fall. Let's consider each of these forces in turn.

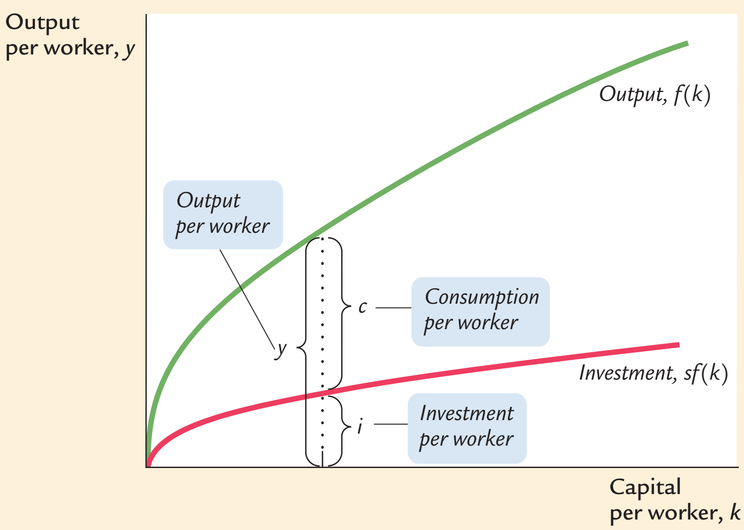
As we have already noted, investment per worker i equals sy. By substituting the production function for y, we can express investment per worker as a function of the capital stock per worker:

$$i = sf(k).$$

This equation relates the existing stock of capital k to the accumulation of new capital i. Figure 8-2 shows this relationship. This figure illustrates how, for any value of k, the amount of output is determined by the production function $f(k)$, and the allocation of that output between consumption and investment is determined by the saving rate s.

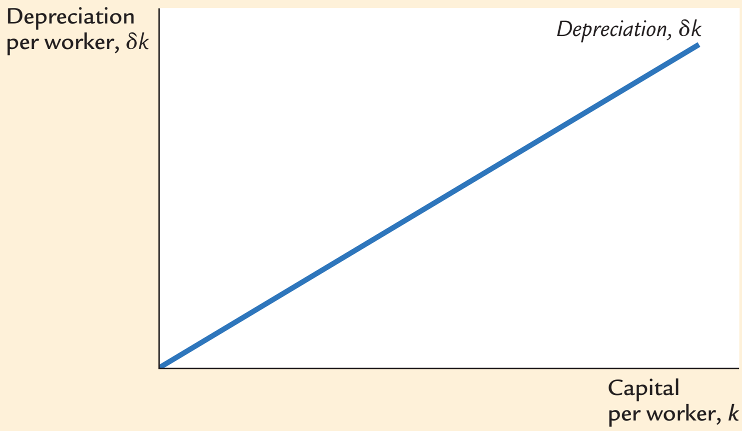
To incorporate depreciation into the model, we assume that a certain fraction δ of the capital stock wears out each year. Here δ (the lowercase Greek letter delta) is called the *depreciation rate*. For example, if capital lasts an average of 25 years, then the depreciation rate is 4 percent per year ($\delta = 0.04$). The amount of capital

FIGURE 8-2



Output, Consumption, and Investment The saving rate s determines the allocation of output between consumption and investment. For any level of capital k, output is $f(k)$, investment is $sf(k)$, and consumption is $f(k) - sf(k)$.

FIGURE 8-3



Depreciation A constant fraction δ of the capital stock wears out every year. Depreciation is therefore proportional to the capital stock.

that depreciates each year is δk. Figure 8-3 shows how the amount of depreciation depends on the capital stock.

We can express the impact of investment and depreciation on the capital stock with this equation:

$$\text{Change in Capital Stock} = \text{Investment} - \text{Depreciation}$$
$$\Delta k = i - \delta k,$$

where Δk is the change in the capital stock between one year and the next. Because investment i equals $sf(k)$, we can write this as

$$\Delta k = sf(k) - \delta k.$$

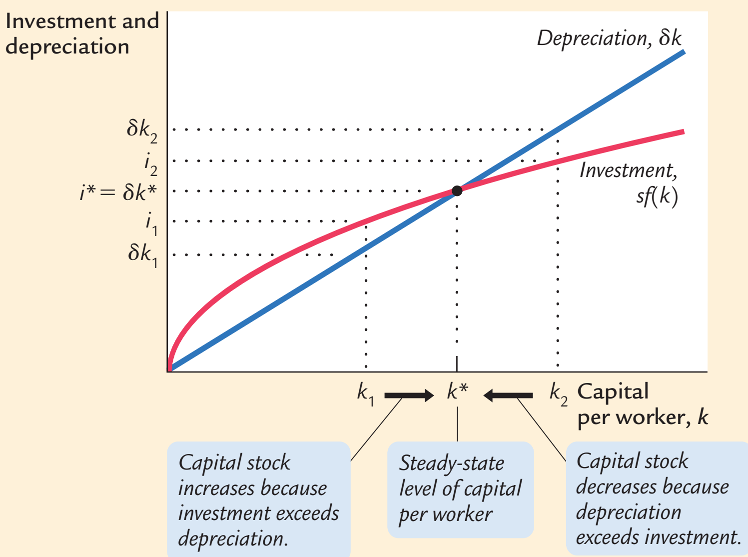
Figure 8-4 graphs the terms of this equation—investment and depreciation—for different levels of the capital stock k. The higher the capital stock, the greater the amounts of output and investment. Yet the higher the capital stock, the greater also the amount of depreciation.

As Figure 8-4 shows, there is a single capital stock k^* at which the amount of investment equals the amount of depreciation. If the economy finds itself at this level of the capital stock, the capital stock will not change because the two forces acting on it—investment and depreciation—just balance. That is, at k^*, $\Delta k = 0$, so the capital stock k and output $f(k)$ are steady over time (rather than growing or shrinking). We therefore call k^* the **steady-state** level of capital.

The steady state is significant for two reasons. As we have just seen, an economy at the steady state will stay there. In addition, and just as important, an economy *not* at the steady state will go there. That is, regardless of the level of capital with which the economy begins, it ends up with the steady-state level of capital. In this sense, *the steady state represents the long-run equilibrium of the economy.*

To see why an economy always ends up at the steady state, suppose that the economy starts with less than the steady-state level of capital, such as level k_1 in Figure 8-4. In this case, the level of investment exceeds the amount of depreciation.

FIGURE 8-4



Investment, Depreciation, and the Steady State The steady-state level of capital k^* is the level at which investment equals depreciation, indicating that the amount of capital will not change over time. Below k^* investment exceeds depreciation, so the capital stock grows. Above k^* investment is less than depreciation, so the capital stock shrinks.

Over time, the capital stock will rise and will continue to rise—along with output $f(k)$—until it approaches the steady state k^*.

Similarly, suppose that the economy starts with more than the steady-state level of capital, such as level k_2. In this case, investment is less than depreciation: capital is wearing out faster than it is being replaced. The capital stock will fall, again approaching the steady-state level. Once the capital stock reaches the steady state, investment equals depreciation, and there is no pressure for the capital stock to either increase or decrease.

Approaching the Steady State: A Numerical Example

Let's use a numerical example to see how the Solow model works and how the economy approaches the steady state. For this example, we assume that the production function is

$$Y = K^{1/2}L^{1/2}.$$

From Chapter 3, you will recognize this as the Cobb–Douglas production function with the capital-share parameter α equal to 1/2. To derive the per-worker production function $f(k)$, divide both sides of the production function by the labor force L:

$$\frac{Y}{L} = \frac{K^{1/2}L^{1/2}}{L}.$$

Rearrange to obtain

$$\frac{Y}{L} = \left(\frac{K}{L}\right)^{1/2}.$$

Because $y = Y/L$ and $k = K/L$, this equation becomes

$$y = k^{1/2},$$

which can also be written as

$$y = \sqrt{k}.$$

This form of the production function states that output per worker equals the square root of the amount of capital per worker.

To complete the example, let's assume that 30 percent of output is saved ($s = 0.3$), that 10 percent of the capital stock depreciates every year ($\delta = 0.1$), and that the economy starts off with 4 units of capital per worker ($k = 4$). Given these numbers, we can now examine what happens to this economy over time.

We begin by looking at the production and allocation of output in the first year, when the economy has 4 units of capital per worker. Here are the steps we follow.

- According to the production function $y = \sqrt{k}$, the 4 units of capital per worker (k) produce 2 units of output per worker (y).
- Because 30 percent of output is saved and invested and 70 percent is consumed, $i = 0.6$ and $c = 1.4$.
- Because 10 percent of the capital stock depreciates, $\delta k = 0.4$.
- With investment of 0.6 and depreciation of 0.4, the change in the capital stock is $\Delta k = 0.2$.

Thus, the economy begins its second year with 4.2 units of capital per worker.

We can do the same calculations for each subsequent year. Table 8-2 shows how the economy progresses. Every year, because investment exceeds depreciation, new capital is added and output grows. Over many years, the economy approaches a steady state with 9 units of capital per worker. In this steady state, investment of 0.9 exactly offsets depreciation of 0.9, so the capital stock and output are no longer growing.

Following the progress of the economy for many years is one way to find the steady-state capital stock, but there is another way that requires fewer calculations. Recall that

$$\Delta k = sf(k) - \delta k.$$

This equation shows how k evolves over time. Because the steady state is (by definition) the value of k at which $\Delta k = 0$, we know that

$$0 = sf(k^*) - \delta k^*,$$

or, equivalently,

$$\frac{k^*}{f(k^*)} = \frac{s}{\delta}.$$

This equation provides a way of finding the steady-state level of capital per worker k^*. Substituting in the numbers and production function from our example, we obtain

$$\frac{k^*}{\sqrt{k^*}} = \frac{0.3}{0.1}.$$

TABLE 8-2

Approaching the Steady State: A Numerical Example

Assumptions: $y = \sqrt{k}$; $s = 0.3$; $\delta = 0.1$; initial $k = 4.0$

Year	k	y	c	i	δk	Δk
1	4.000	2.000	1.400	0.600	0.400	0.200
2	4.200	2.049	1.435	0.615	0.420	0.195
3	4.395	2.096	1.467	0.629	0.440	0.189
4	4.584	2.141	1.499	0.642	0.458	0.184
5	4.768	2.184	1.529	0.655	0.477	0.178
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10	5.602	2.367	1.657	0.710	0.560	0.150
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25	7.321	2.706	1.894	0.812	0.732	0.080
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100	8.962	2.994	2.096	0.898	0.896	0.002
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∞	9.000	3.000	2.100	0.900	0.900	0.000

Now square both sides of this equation to find

$$k^* = 9.$$

The steady-state capital stock is 9 units per worker. This result confirms the calculation of the steady state in Table 8-2.

CASE STUDY

The Miracle of Japanese and German Growth

Japan and Germany are two success stories of economic growth. Although today they are economic superpowers, in 1945 the economies of both countries were in shambles. World War II had destroyed much of their capital stocks. In the decades after the war, however, these two countries experienced some of the most rapid growth rates on record. Between 1948 and 1972, output per person grew at 8.2 percent per year in Japan and 5.7 percent per year in Germany, compared to only 2.2 percent per year in the United States.

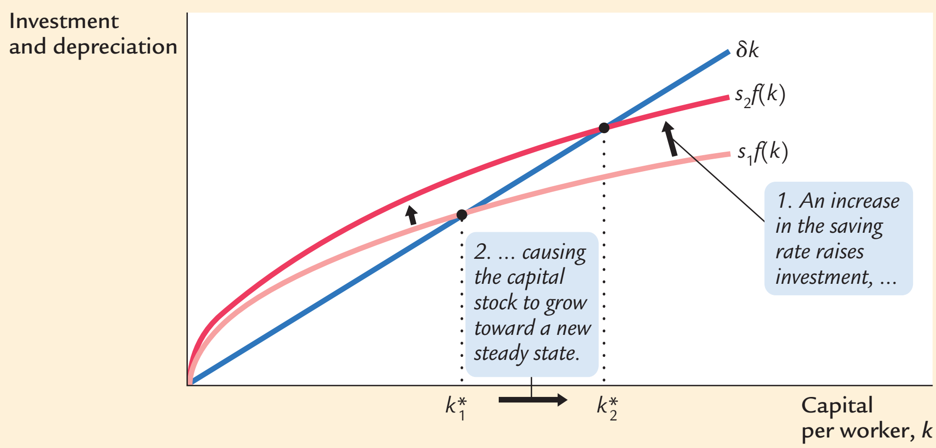
Are the postwar experiences of Japan and Germany so surprising from the standpoint of the Solow growth model? Consider an economy in steady state. Now suppose that a war destroys some of the capital stock. (That is, suppose the capital stock drops from k^* to k_1 in Figure 8-4.) Not surprisingly, the level of output falls immediately. But if the saving rate—the fraction of output devoted to saving and investment—is unchanged, the economy will then experience a period of high growth. Output grows because, at the lower capital stock, more capital is added by investment than is removed by depreciation. This high growth continues until the economy approaches its former steady state. Hence, although destroying part of the capital stock immediately reduces output, it is followed by higher-than-normal growth. The "miracle" of rapid growth in Japan and Germany, as it is often described in the business press, is what the Solow model predicts for countries in which war has greatly reduced the capital stock. ■

How Saving Affects Growth

The explanation of Japanese and German growth after World War II is not quite as simple as suggested in the preceding Case Study. Another relevant fact is that both Japan and Germany save and invest a higher fraction of their output than does the United States. To understand more fully the international differences in economic performance, we must consider the effects of different saving rates.

Consider what happens to an economy when its saving rate increases. Figure 8-5 shows such a change. The economy is assumed to begin in a steady state

FIGURE 8-5



An Increase in the Saving Rate An increase in the saving rate s implies that the amount of investment for any given capital stock is higher. It therefore shifts the saving function upward. At the initial steady state k_1^*, investment now exceeds depreciation. The capital stock rises until the economy reaches a new steady state k_2^* with more capital and output.

with saving rate s_1 and capital stock k_1^*. When the saving rate increases from s_1 to s_2, the $sf(k)$ curve shifts upward. At the initial saving rate s_1 and the initial capital stock k_1^*, the amount of investment just offsets the amount of depreciation. Immediately after the saving rate rises, investment is higher, but the capital stock and depreciation are unchanged. Therefore, investment exceeds depreciation. The capital stock gradually rises until the economy reaches the new steady state k_2^*, which has a higher capital stock and a higher level of output than the old steady state.

The Solow model shows that the saving rate is a key determinant of the steady-state capital stock. *If the saving rate is high, the economy will have a large capital stock and a high level of output in the steady state. If the saving rate is low, the economy will have a small capital stock and a low level of output in the steady state.* This conclusion sheds light on many discussions of fiscal policy. As we saw in Chapter 3, a government budget deficit can reduce national saving and crowd out investment. Now we can see that the long-run consequences of a reduced saving rate are a lower capital stock and lower national income. This is why many economists are critical of persistent budget deficits.

What does the Solow model say about the relationship between saving and economic growth? Higher saving leads to faster growth in the Solow model, but only temporarily. An increase in the rate of saving raises growth only until the economy reaches the new steady state. If the economy maintains a high saving rate, it will maintain a large capital stock and a high level of output, but it will not maintain a high rate of growth forever. Policies that alter the steady-state growth rate of income per person are said to have a *growth effect*; we will see examples of such policies in the next chapter. By contrast, a higher saving rate is said to have a *level effect*, because only the level of income per person—not its growth rate—is influenced by the saving rate in the steady state.

Now that we understand how saving and growth interact, we can more fully explain the impressive economic performance of Germany and Japan after World War II. Not only were their initial capital stocks low because of the war, but their steady-state capital stocks were also high because of their high saving rates. Both of these facts help explain the rapid growth of these two countries in the 1950s and 1960s.

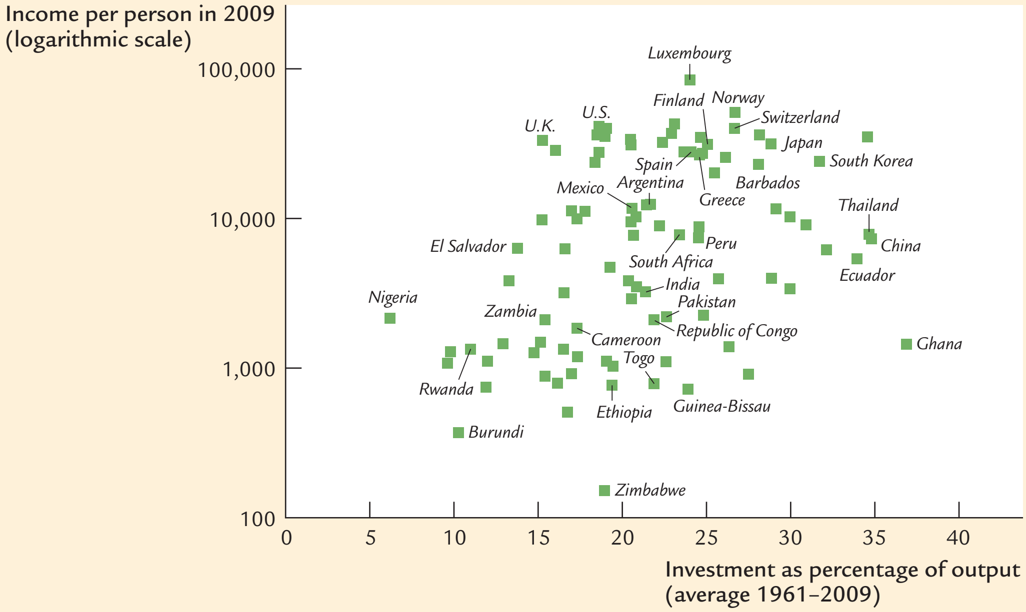
CASE STUDY

Saving and Investment Around the World

We started this chapter with an important question: Why are some countries so rich while others are mired in poverty? Our analysis has taken us a step closer to the answer. According to the Solow model, if a nation devotes a large fraction of its income to saving and investment, it will have a high steady-state capital stock and a high level of income. If a nation saves and invests only a small fraction of its income, its steady-state capital and income will be low.

Let's now look at some data to see if this theoretical result in fact helps explain the large international variation in standards of living. Figure 8-6 is a scatterplot

FIGURE 8-6



International Evidence on Investment Rates and Income per Person This scatterplot shows the experience of about 100 countries, each represented by a single point. The horizontal axis shows the country's rate of investment, and the vertical axis shows the country's income per person. High investment is associated with high income per person, as the Solow model predicts. The correlation between these two variables is 0.25.

Source: Alan Heston, Robert Summers, and Bettina Aten, Penn World Table Version 7.0, Center for International Comparisons of Production, Income, and Prices at the University of Pennsylvania, May 2011.

of data from about 100 countries. (The figure includes most of the world's economies. It excludes major oil-producing countries and countries that were communist during much of this period, because their experiences are explained by their special circumstances.) The data show a positive relationship between the fraction of output devoted to investment and the level of income per person. That is, countries with high rates of investment, such as South Korea and Japan, usually have high incomes, whereas countries with low rates of investment, such as Nigeria and Burundi, have low incomes. Thus, the data are consistent with the Solow model's prediction that the investment rate is a key determinant of whether a country is rich or poor.

The positive correlation shown in this figure is an important fact, but it raises as many questions as it resolves. One might naturally ask, why do rates of saving and investment vary so much from country to country? There are many potential answers, such as tax policy, retirement patterns, the development of financial markets, and cultural differences. In addition, political stability may play a role: not surprisingly, rates of saving and investment tend to be low in countries with frequent wars, revolutions, and coups. Saving and investment also tend to be low in countries with poor political institutions, as measured by estimates of official corruption. A final interpretation of the evidence in Figure 8-6 is reverse causation: perhaps high levels of income somehow foster high rates of saving and investment. Unfortunately, there is no consensus among economists about which of the many possible explanations is most important.

The association between investment rates and income per person is an important clue as to why some countries are rich and others poor, but it is not the whole story. The correlation between these two variables is far from perfect. There must be other determinants of living standards beyond saving and investment. Later in this chapter and in the next one, we return to the international differences in income per person to see what other variables enter the picture. ■

8-2 The Golden Rule Level of Capital

So far, we have used the Solow model to examine how an economy's rate of saving and investment determines its steady-state levels of capital and income. This analysis might lead you to think that higher saving is always a good thing because it always leads to greater income. Yet suppose a nation had a saving rate of 100 percent. That would lead to the largest possible capital stock and the largest possible income. But if all of this income is saved and none is ever consumed, what good is it?

This section uses the Solow model to discuss the optimal amount of capital accumulation from the standpoint of economic well-being. In the next chapter, we discuss how government policies influence a nation's saving rate. But first, in this section, we present the theory behind these policy decisions.

Comparing Steady States

To keep our analysis simple, let's assume that a policymaker can set the economy's saving rate at any level. By setting the saving rate, the policymaker determines the economy's steady state. What steady state should the policymaker choose?

The policymaker's goal is to maximize the well-being of the individuals who make up the society. Individuals themselves do not care about the amount of capital in the economy or even the amount of output. They care about the amount of goods and services they can consume. Thus, a benevolent policymaker would want to choose the steady state with the highest level of consumption. The steady-state value of k that maximizes consumption is called the **Golden Rule level of capital** and is denoted k^*_{gold}.[2]

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[2]Edmund Phelps, "The Golden Rule of Accumulation: A Fable for Growthmen," *American Economic Review* 51 (September 1961): 638–643.

How can we tell whether an economy is at the Golden Rule level? To answer this question, we must first determine steady-state consumption per worker. Then we can see which steady state provides the most consumption.

To find steady-state consumption per worker, we begin with the national income accounts identity

$$y = c + i$$

and rearrange it as

$$c = y - i.$$

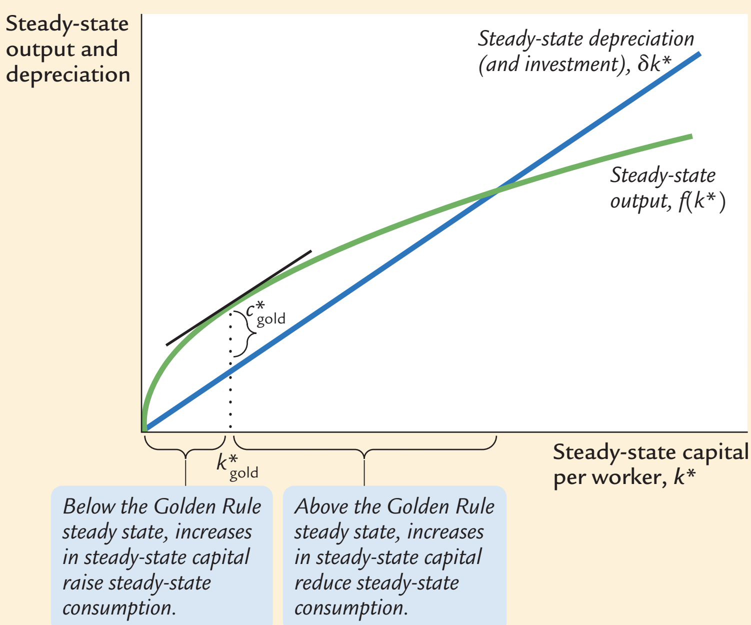
Consumption is output minus investment. Because we want to find steady-state consumption, we substitute steady-state values for output and investment. Steady-state output per worker is $f(k^*)$, where k^* is the steady-state capital stock per worker. Furthermore, because the capital stock is not changing in the steady state, investment equals depreciation δk^*. Substituting $f(k^*)$ for y and δk^* for i, we can write steady-state consumption per worker as

$$c^* = f(k^*) - \delta k^*.$$

According to this equation, steady-state consumption is what's left of steady-state output after paying for steady-state depreciation. This equation shows that an increase in steady-state capital has two opposing effects on steady-state consumption. On the one hand, more capital means more output. On the other hand, more capital also means that more output must be used to replace capital that is wearing out.

Figure 8-7 graphs steady-state output and steady-state depreciation as a function of the steady-state capital stock. Steady-state consumption is the gap between

FIGURE 8-7



Steady-State Consumption The economy's output is used for consumption or investment. In the steady state, investment equals depreciation. Therefore, steady-state consumption is the difference between output $f(k^*)$ and depreciation δk^*. Steady-state consumption is maximized at the Golden Rule steady state. The Golden Rule capital stock is denoted k^*_{gold}, and the Golden Rule level of consumption is denoted c^*_{gold}.

output and depreciation. This figure shows that there is one level of the capital stock—the Golden Rule level k^*_{gold}—that maximizes consumption.

When comparing steady states, we must keep in mind that higher levels of capital affect both output and depreciation. If the capital stock is below the Golden Rule level, an increase in the capital stock raises output more than depreciation, so consumption rises. In this case, the production function is steeper than the δk^* line, so the gap between these two curves—which equals consumption—grows as k^* rises. By contrast, if the capital stock is above the Golden Rule level, an increase in the capital stock reduces consumption, because the increase in output is smaller than the increase in depreciation. In this case, the production function is flatter than the δk^* line, so the gap between the curves—consumption—shrinks as k^* rises. At the Golden Rule level of capital, the production function and the δk^* line have the same slope, and consumption is at its greatest level.

We can now derive a simple condition that characterizes the Golden Rule level of capital. Recall that the slope of the production function is the marginal product of capital MPK. The slope of the δk^* line is δ. Because these two slopes are equal at k^*_{gold}, the Golden Rule is described by the equation

$$MPK = \delta.$$

At the Golden Rule level of capital, the marginal product of capital equals the depreciation rate.

To make the point somewhat differently, suppose that the economy starts at some steady-state capital stock k^* and that the policymaker is considering increasing the capital stock to $k^* + 1$. The amount of extra output from this increase in capital would be $f(k^* + 1) - f(k^*)$, the marginal product of capital MPK. The amount of extra depreciation from having 1 more unit of capital is the depreciation rate δ. Thus, the net effect of this extra unit of capital on consumption is $MPK - \delta$. If $MPK - \delta > 0$, then increases in capital increase consumption, so k^* must be below the Golden Rule level. If $MPK - \delta < 0$, then increases in capital decrease consumption, so k^* must be above the Golden Rule level. Therefore, the following condition describes the Golden Rule:

$$MPK - \delta = 0.$$

At the Golden Rule level of capital, the marginal product of capital net of depreciation ($MPK - \delta$) equals zero. As we will see, a policymaker can use this condition to find the Golden Rule capital stock for an economy.[3]

Keep in mind that the economy does not automatically gravitate toward the Golden Rule steady state. If we want any particular steady-state capital stock, such as the Golden Rule, we need a particular saving rate to support it. Figure 8-8 shows

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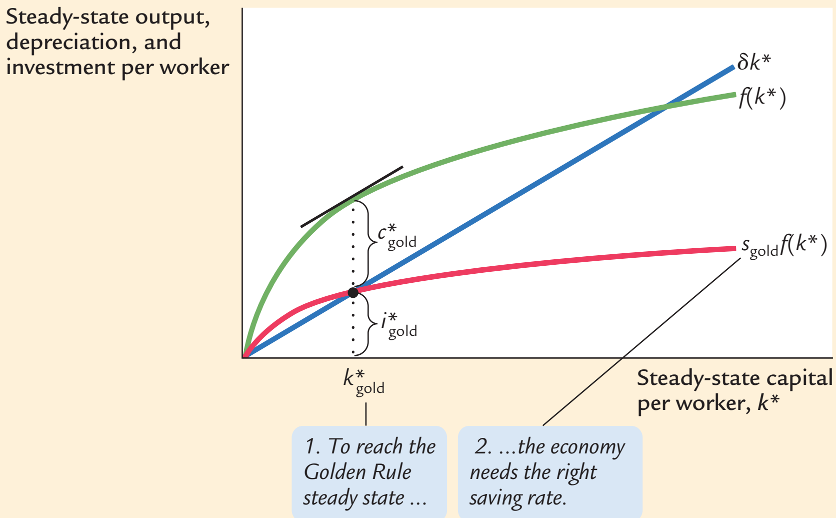
[3] *Mathematical note:* Another way to derive the condition for the Golden Rule uses a bit of calculus. Recall that $c^* = f(k^*) - \delta k^*$. To find the k^* that maximizes c^*, differentiate to find $dc^*/dk^* = f'(k^*) - \delta$ and set this derivative equal to zero. Noting that $f'(k^*)$ is the marginal product of capital, we obtain the Golden Rule condition in the text.

FIGURE 8-8



The Saving Rate and the Golden Rule There is only one saving rate that produces the Golden Rule level of capital k^*_{gold}. Any change in the saving rate would shift the $sf(k)$ curve and would move the economy to a steady state with a lower level of consumption.

the steady state if the saving rate is set to produce the Golden Rule level of capital. If the saving rate is higher than the one used in this figure, the steady-state capital stock will be too high. If the saving rate is lower, the steady-state capital stock will be too low. In either case, steady-state consumption will be lower than it is at the Golden Rule steady state.

Finding the Golden Rule Steady State: A Numerical Example

Consider the decision of a policymaker choosing a steady state in the following economy. The production function is the same as in our earlier example:

$$y = \sqrt{k}.$$

Output per worker is the square root of capital per worker. Depreciation δ is again 10 percent of capital. This time, the policymaker chooses the saving rate s and thus the economy's steady state.

To see the outcomes available to the policymaker, recall that the following equation holds in the steady state:

$$\frac{k^*}{f(k^*)} = \frac{s}{\delta}$$

In this economy, this equation becomes

$$\frac{k^*}{\sqrt{k^*}} = \frac{s}{0.1}.$$

Squaring both sides of this equation yields a solution for the steady-state capital stock. We find

$$k^* = 100s^2.$$

Using this result, we can compute the steady-state capital stock for any saving rate.

Table 8-3 presents calculations showing the steady states that result from various saving rates in this economy. We see that higher saving leads to a higher capital stock, which in turn leads to higher output and higher depreciation. Steady-state consumption, the difference between output and depreciation, first rises with higher saving rates and then declines. Consumption is highest when the saving rate is 0.5. Hence, a saving rate of 0.5 produces the Golden Rule steady state.

Recall that another way to identify the Golden Rule steady state is to find the capital stock at which the net marginal product of capital ($MPK - \delta$) equals zero. For this production function, the marginal product is[4]

$$MPK = \frac{1}{2\sqrt{k}}.$$

Using this formula, the last two columns of Table 8-3 present the values of MPK and $MPK - \delta$ in the different steady states. Note that the net marginal product

TABLE 8-3

Finding the Golden Rule Steady State: A Numerical Example

Assumptions: $y = \sqrt{k}$; $\delta = 0.1$

s	k^*	y^*	δk^*	c^*	MPK	$MPK - \delta$
0.0	0.0	0.0	0.0	0.0	∞	∞
0.1	1.0	1.0	0.1	0.9	0.500	0.400
0.2	4.0	2.0	0.4	1.6	0.250	0.150
0.3	9.0	3.0	0.9	2.1	0.167	0.067
0.4	16.0	4.0	1.6	2.4	0.125	0.025
0.5	**25.0**	**5.0**	**2.5**	**2.5**	**0.100**	**0.000**
0.6	36.0	6.0	3.6	2.4	0.083	–0.017
0.7	49.0	7.0	4.9	2.1	0.071	–0.029
0.8	64.0	8.0	6.4	1.6	0.062	–0.038
0.9	81.0	9.0	8.1	0.9	0.056	–0.044
1.0	100.0	10.0	10.0	0.0	0.050	–0.050

[4] *Mathematical note:* To derive this formula, note that the marginal product of capital is the derivative of the production function with respect to k.

of capital is exactly zero when the saving rate is at its Golden Rule value of 0.5. Because of diminishing marginal product, the net marginal product of capital is greater than zero whenever the economy saves less than this amount, and it is less than zero whenever the economy saves more.

This numerical example confirms that the two ways of finding the Golden Rule steady state—looking at steady-state consumption or looking at the marginal product of capital—give the same answer. If we want to know whether an actual economy is currently at, above, or below its Golden Rule capital stock, the second method is usually more convenient, because it is relatively straightforward to estimate the marginal product of capital. By contrast, evaluating an economy with the first method requires estimates of steady-state consumption at many different saving rates; such information is harder to obtain. Thus, when we apply this kind of analysis to the U.S. economy in the next chapter, we will evaluate U.S. saving by examining the marginal product of capital. Before engaging in that policy analysis, however, we need to proceed further in our development and understanding of the Solow model.

The Transition to the Golden Rule Steady State

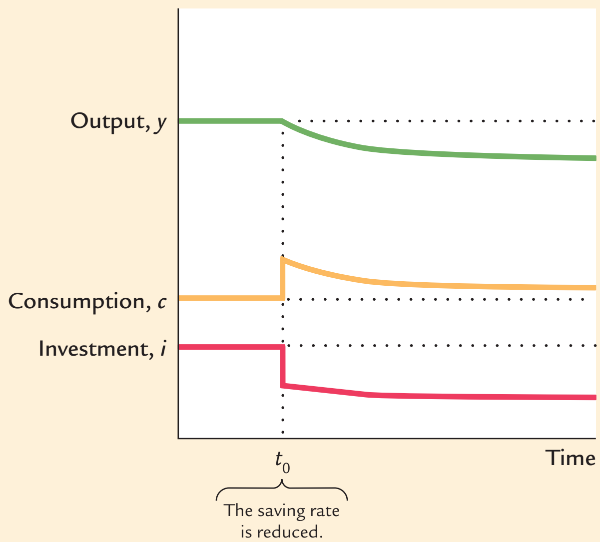
Let's now make our policymaker's problem more realistic. So far, we have been assuming that the policymaker can simply choose the economy's steady state and jump there immediately. In this case, the policymaker would choose the steady state with the highest consumption—the Golden Rule steady state. But now suppose that the economy has reached a steady state other than the Golden Rule. What happens to consumption, investment, and capital when the economy makes the transition between steady states? Might the impact of the transition deter the policymaker from trying to achieve the Golden Rule?

We must consider two cases: the economy might begin with more capital than in the Golden Rule steady state, or with less. It turns out that the two cases offer very different problems for policymakers. (As we will see in the next chapter, the second case—too little capital—describes most actual economies, including that of the United States.)

Starting With Too Much Capital We first consider the case in which the economy begins at a steady state with more capital than it would have in the Golden Rule steady state. In this case, the policymaker should pursue policies aimed at reducing the rate of saving in order to reduce the capital stock. Suppose that these policies succeed and that at some point—call it time t_0—the saving rate falls to the level that will eventually lead to the Golden Rule steady state.

Figure 8-9 shows what happens to output, consumption, and investment when the saving rate falls. The reduction in the saving rate causes an immediate increase in consumption and a decrease in investment. Because investment and depreciation were equal in the initial steady state, investment will now be less than depreciation, which means the economy is no longer in a steady state. Gradually, the capital stock falls, leading to reductions in output, consumption, and investment. These variables

FIGURE 8-9



Reducing Saving When Starting With More Capital Than in the Golden Rule Steady State This figure shows what happens over time to output, consumption, and investment when the economy begins with more capital than the Golden Rule level and the saving rate is reduced. The reduction in the saving rate (at time t_0) causes an immediate increase in consumption and an equal decrease in investment. Over time, as the capital stock falls, output, consumption, and investment fall together. Because the economy began with too much capital, the new steady state has a higher level of consumption than the initial steady state.

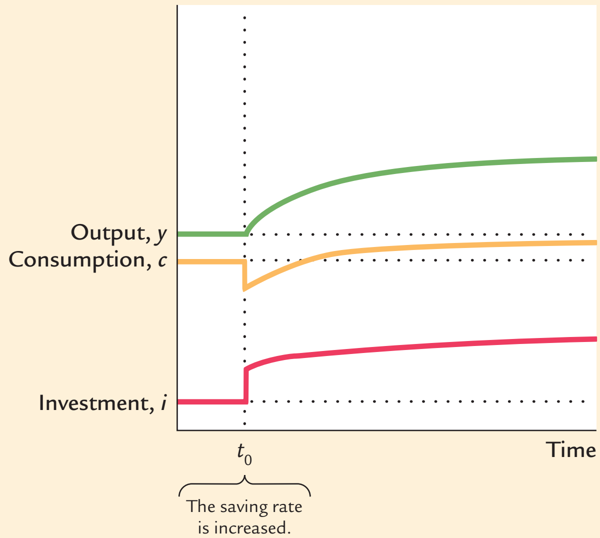
continue to fall until the economy reaches the new steady state. Because we are assuming that the new steady state is the Golden Rule steady state, consumption must be higher than it was before the change in the saving rate, even though output and investment are lower.

Note that, compared to the old steady state, consumption is higher not only in the new steady state but also along the entire path to it. When the capital stock exceeds the Golden Rule level, reducing saving is clearly a good policy, for it increases consumption at every point in time.

Starting With Too Little Capital When the economy begins with less capital than in the Golden Rule steady state, the policymaker must raise the saving rate to reach the Golden Rule. Figure 8-10 shows what happens. The increase in the saving rate at time t_0 causes an immediate fall in consumption and a rise in investment. Over time, higher investment causes the capital stock to rise. As capital accumulates, output, consumption, and investment gradually increase, eventually approaching the new steady-state levels. Because the initial steady state was below the Golden Rule, the increase in saving eventually leads to a higher level of consumption than that which prevailed initially.

Does the increase in saving that leads to the Golden Rule steady state raise economic welfare? Eventually it does, because the new steady-state level of consumption is higher than the initial level. But achieving that new steady state requires an initial period of reduced consumption. Note the contrast to the case in which the economy begins above the Golden Rule. *When the economy begins above the Golden Rule, reaching the Golden Rule produces higher consumption at all points in time. When the economy begins below the Golden Rule, reaching the Golden Rule requires initially reducing consumption to increase consumption in the future.*

FIGURE 8-10



Increasing Saving When Starting With Less Capital Than in the Golden Rule Steady State This figure shows what happens over time to output, consumption, and investment when the economy begins with less capital than the Golden Rule level and the saving rate is increased. The increase in the saving rate (at time t_0) causes an immediate drop in consumption and an equal jump in investment. Over time, as the capital stock grows, output, consumption, and investment increase together. Because the economy began with less capital than the Golden Rule level, the new steady state has a higher level of consumption than the initial steady state.

When deciding whether to try to reach the Golden Rule steady state, policymakers have to take into account that current consumers and future consumers are not always the same people. Reaching the Golden Rule achieves the highest steady-state level of consumption and thus benefits future generations. But when the economy is initially below the Golden Rule, reaching the Golden Rule requires raising investment and thus lowering the consumption of current generations. Thus, when choosing whether to increase capital accumulation, the policymaker faces a tradeoff among the welfare of different generations. A policymaker who cares more about current generations than about future ones may decide not to pursue policies to reach the Golden Rule steady state. By contrast, a policymaker who cares about all generations equally will choose to reach the Golden Rule. Even though current generations will consume less, an infinite number of future generations will benefit by moving to the Golden Rule.

Thus, optimal capital accumulation depends crucially on how we weigh the interests of current and future generations. The biblical Golden Rule tells us, "Do unto others as you would have them do unto you." If we heed this advice, we give all generations equal weight. In this case, it is optimal to reach the Golden Rule level of capital—which is why it is called the "Golden Rule."

8-3 Population Growth

The basic Solow model shows that capital accumulation, by itself, cannot explain sustained economic growth: high rates of saving lead to high growth temporarily, but the economy eventually approaches a steady state in which capital and output are constant. To explain the sustained economic growth that we observe in most parts of the world, we must expand the Solow model to incorporate the other two sources of economic growth—population growth and technological progress. In this section we add population growth to the model.

Instead of assuming that the population is fixed, as we did in Sections 8-1 and 8-2, we now suppose that the population and the labor force grow at a constant rate n. For example, the U.S. population grows about 1 percent per year, so $n = 0.01$. This means that if 150 million people are working one year, then 151.5 million (1.01×150) are working the next year, and 153.015 million (1.01×151.5) the year after that, and so on.

The Steady State With Population Growth

How does population growth affect the steady state? To answer this question, we must discuss how population growth, along with investment and depreciation, influences the accumulation of capital per worker. As we noted before, investment raises the capital stock, and depreciation reduces it. But now there is a third force acting to change the amount of capital per worker: the growth in the number of workers causes capital per worker to fall.

We continue to let lowercase letters stand for quantities per worker. Thus, $k = K/L$ is capital per worker, and $y = Y/L$ is output per worker. Keep in mind, however, that the number of workers is growing over time.

The change in the capital stock per worker is

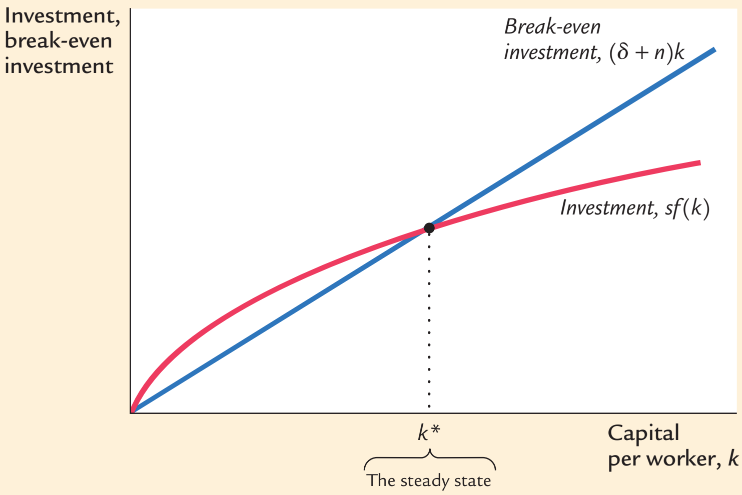
$$\Delta k = i - (\delta + n)k.$$

This equation shows how investment, depreciation, and population growth influence the per-worker capital stock. Investment increases k, whereas depreciation and population growth decrease k. We saw this equation earlier in this chapter for the special case of a constant population ($n = 0$).

We can think of the term $(\delta + n)k$ as defining *break-even investment*—the amount of investment necessary to keep the capital stock per worker constant. Break-even investment includes the depreciation of existing capital, which equals δk. It also includes the amount of investment necessary to provide new workers with capital. The amount of investment necessary for this purpose is nk, because there are n new workers for each existing worker and because k is the amount of capital for each worker. The equation shows that population growth reduces the accumulation of capital per worker much the way depreciation does. Depreciation reduces k by wearing out the capital stock, whereas population growth reduces k by spreading the capital stock more thinly among a larger population of workers.[5]

[5] *Mathematical note:* Formally deriving the equation for the change in k requires a bit of calculus. Note that the change in k per unit of time is $dk/dt = d(K/L)/dt$. After applying the standard rules of calculus, we can write this as $dk/dt = (1/L)(dK/dt) - (K/L^2)(dL/dt)$. Now use the following facts to substitute in this equation: $dK/dt = I - \delta K$ and $(dL/dt)/L = n$. After a bit of manipulation, this produces the equation in the text.

FIGURE 8-11



Population Growth in the Solow Model Depreciation and population growth are two reasons the capital stock per worker shrinks. If n is the rate of population growth and δ is the rate of depreciation, then $(\delta + n)k$ is *break-even investment*—the amount of investment necessary to keep constant the capital stock per worker k. For the economy to be in a steady state, investment $sf(k)$ must offset the effects of depreciation and population growth $(\delta + n)k$. This is represented by the crossing of the two curves.

Our analysis with population growth now proceeds much as it did previously. First, we substitute $sf(k)$ for i. The equation can then be written as

$$\Delta k = sf(k) - (\delta + n)k.$$

To see what determines the steady-state level of capital per worker, we use Figure 8-11, which extends the analysis of Figure 8-4 to include the effects of population growth. An economy is in a steady state if capital per worker k is unchanging. As before, we designate the steady-state value of k as k^*. If k is less than k^*, investment is greater than break-even investment, so k rises. If k is greater than k^*, investment is less than break-even investment, so k falls.

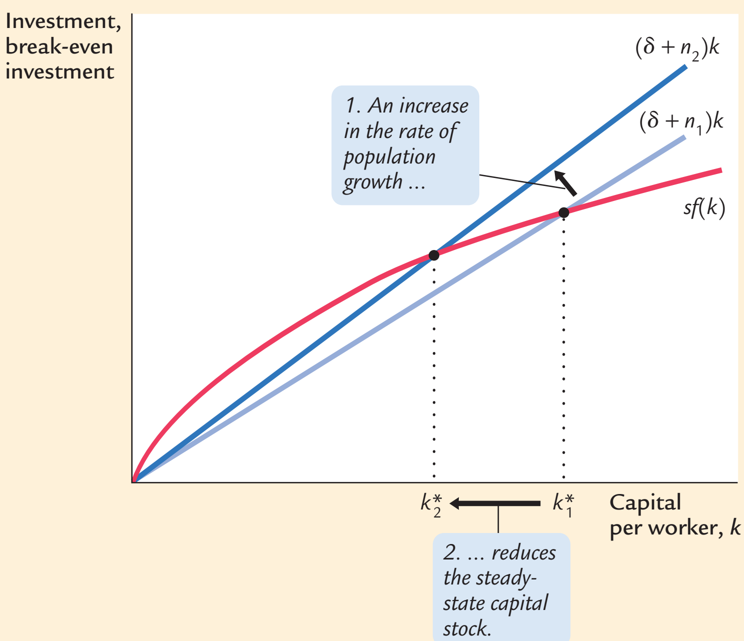
In the steady state, the positive effect of investment on the capital stock per worker exactly balances the negative effects of depreciation and population growth. That is, at k^*, $\Delta k = 0$ and $i^* = \delta k^* + nk^*$. Once the economy is in the steady state, investment has two purposes. Some of it (δk^*) replaces the depreciated capital, and the rest (nk^*) provides the new workers with the steady-state amount of capital.

The Effects of Population Growth

Population growth alters the basic Solow model in three ways. First, it brings us closer to explaining sustained economic growth. In the steady state with population growth, capital per worker and output per worker are constant. Because the number of workers is growing at rate n, however, *total* capital and *total* output must also be growing at rate n. Hence, although population growth cannot explain sustained growth in the standard of living (because output per worker is constant in the steady state), it can help explain sustained growth in total output.

Second, population growth gives us another explanation for why some countries are rich and others are poor. Consider the effects of an increase in population growth. Figure 8-12 shows that an increase in the rate of population growth from n_1 to n_2

FIGURE 8-12



The Impact of Population Growth An increase in the rate of population growth from n_1 to n_2 shifts the line representing population growth and depreciation upward. The new steady state k_2^* has a lower level of capital per worker than the initial steady state k_1^*. Thus, the Solow model predicts that economies with higher rates of population growth will have lower levels of capital per worker and therefore lower incomes.

reduces the steady-state level of capital per worker from k_1^* to k_2^*. Because k^* is lower and because $y^* = f(k^*)$, the level of output per worker y^* is also lower. Thus, the Solow model predicts that countries with higher population growth will have lower levels of GDP per person. Notice that a change in the population growth rate, like a change in the saving rate, has a level effect on income per person but does not affect the steady-state growth rate of income per person.

Finally, population growth affects our criterion for determining the Golden Rule (consumption-maximizing) level of capital. To see how this criterion changes, note that consumption per worker is

$$c = y - i.$$

Because steady-state output is $f(k^*)$ and steady-state investment is $(\delta + n)k^*$, we can express steady-state consumption as

$$c^* = f(k^*) - (\delta + n)k^*.$$

Using an argument largely the same as before, we conclude that the level of k^* that maximizes consumption is the one at which

$$MPK = \delta + n,$$

or equivalently,

$$MPK - \delta = n.$$

In the Golden Rule steady state, the marginal product of capital net of depreciation equals the rate of population growth.

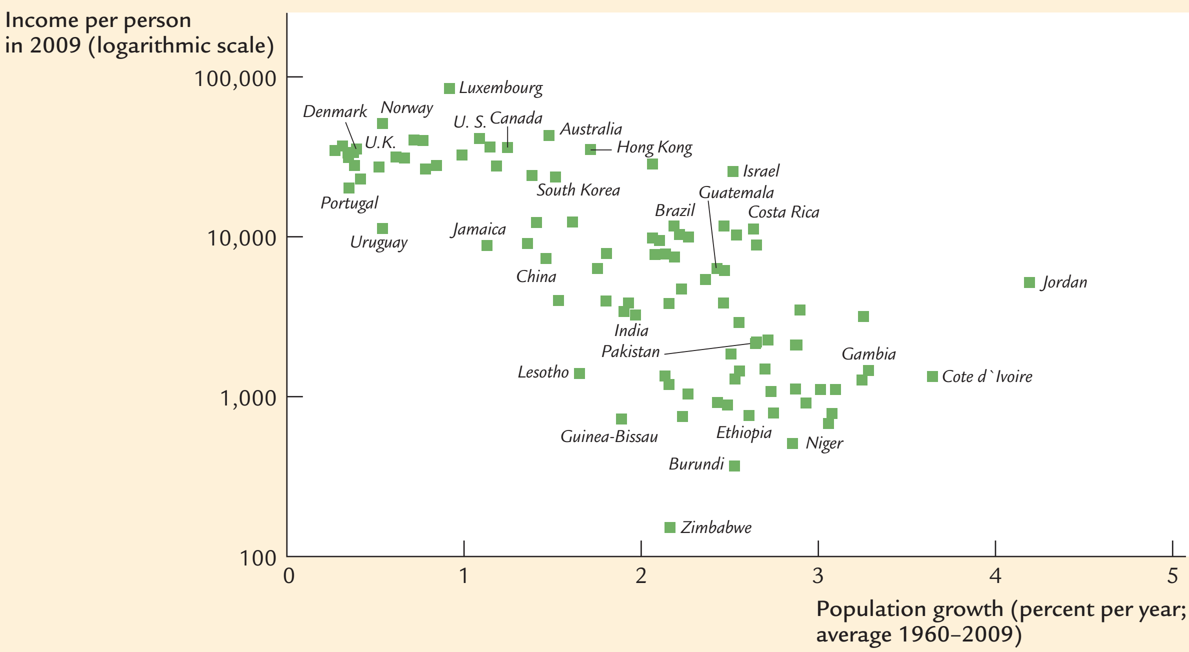
CASE STUDY

Population Growth Around the World

Let's return now to the question of why standards of living vary so much around the world. The analysis we have just completed suggests that population growth may be one of the answers. According to the Solow model, a nation with a high rate of population growth will have a low steady-state capital stock per worker and thus also a low level of income per worker. In other words, high population growth tends to impoverish a country because it is hard to maintain a high level of capital per worker when the number of workers is growing quickly. To see whether the evidence supports this conclusion, we again look at cross-country data.

Figure 8-13 is a scatterplot of data for the same countries examined in the previous Case Study (and in Figure 8-6). The figure shows that countries with high rates of population growth tend to have low levels of income per person. The international evidence is consistent with our model's prediction that the rate of population growth is one determinant of a country's standard of living.

FIGURE 8-13



International Evidence on Population Growth and Income per Person This figure is a scatterplot of data from about 100 countries. It shows that countries with high rates of population growth tend to have low levels of income per person, as the Solow model predicts. The correlation between these variables is –0.74.

Source: Alan Heston, Robert Summers, and Bettina Aten, Penn World Table Version 7.0, Center for International Comparisons of Production, Income, and Prices at the University of Pennsylvania, May 2011.

This conclusion is not lost on policymakers. Those trying to pull the world's poorest nations out of poverty, such as the advisers sent to developing nations by the World Bank, often advocate reducing fertility by increasing education about birth-control methods and expanding women's job opportunities. Toward the same end, China has followed the totalitarian policy of allowing only one child for most urban couples. These policies to reduce population growth should, if the Solow model is right, raise income per person in the long run.

In interpreting the cross-country data, however, it is important to keep in mind that correlation does not imply causation. The data show that low population growth is typically associated with high levels of income per person, and the Solow model offers one possible explanation for this fact, but other explanations are also possible. It is conceivable that high income encourages low population growth, perhaps because birth-control techniques are more readily available in richer countries. The international data can help us evaluate a theory of growth, such as the Solow model, because they show us whether the theory's predictions are borne out in the world. But often more than one theory can explain the same facts. ■

Alternative Perspectives on Population Growth

The Solow growth model highlights the interaction between population growth and capital accumulation. In this model, high population growth reduces output per worker because rapid growth in the number of workers forces the capital stock to be spread more thinly, so in the steady state, each worker is equipped with less capital. The model omits some other potential effects of population growth. Here we consider two—one emphasizing the interaction of population with natural resources, the other emphasizing the interaction of population with technology.

The Malthusian Model In his book *An Essay on the Principle of Population as It Affects the Future Improvement of Society,* the early economist Thomas Robert Malthus (1766–1834) offered what may be history's most chilling forecast. Malthus argued that an ever-increasing population would continually strain society's ability to provide for itself. Mankind, he predicted, would forever live in poverty.

Malthus began by noting that "food is necessary to the existence of man" and that "the passion between the sexes is necessary and will remain nearly in its present state." He concluded that "the power of population is infinitely greater than the power in the earth to produce subsistence for man." According to Malthus, the only check on population growth was "misery and vice." Attempts by charities or governments to alleviate poverty were counterproductive, he argued, because they merely allowed the poor to have more children, placing even greater strains on society's productive capabilities.

The Malthusian model may have described the world when Malthus lived, but its prediction that mankind would remain in poverty forever has proven very wrong. The world population has increased about sixfold over the past two centuries, but average living standards are much higher. Because of economic growth, chronic hunger and malnutrition are less common now than they were in Malthus's day. Famines occur from time to time, but they are more often the result of unequal income distribution or political instability than the inadequate production of food.

Malthus failed to foresee that growth in mankind's ingenuity would more than offset the effects of a larger population. Pesticides, fertilizers, mechanized farm equipment, new crop varieties, and other technological advances that Malthus never imagined have allowed each farmer to feed ever-greater numbers of people. Even with more mouths to feed, fewer farmers are necessary because each farmer is so productive. Today, fewer than 2 percent of Americans work on farms, producing enough food to feed the nation and some excess to export as well.

In addition, although the "passion between the sexes" is just as strong now as it was in Malthus's day, the link between passion and population growth that Malthus assumed has been broken by modern birth control. Many advanced nations, such as those in western Europe, are now experiencing fertility below replacement rates. Over the next century, shrinking populations may be more likely than rapidly expanding ones. There is now little reason to think that an ever-expanding population will overwhelm food production and doom mankind to poverty.[6]

The Kremerian Model While Malthus saw population growth as a threat to rising living standards, economist Michael Kremer has suggested that world population growth is a key driver of advancing economic prosperity. If there are more people, Kremer argues, then there are more scientists, inventors, and engineers to contribute to innovation and technological progress.

As evidence for this hypothesis, Kremer begins by noting that over the broad span of human history, world growth rates have increased together with world population. For example, world growth was more rapid when the world population was 1 billion (which occurred around the year 1800) than it was when the population was only 100 million (around 500 B.C.). This fact is consistent with the hypothesis that having more people induces more technological progress.

Kremer's second, more compelling piece of evidence comes from comparing regions of the world. The melting of the polar ice caps at the end of the ice age around 10,000 B.C. flooded the land bridges and separated the world into several distinct regions that could not communicate with one another for thousands of years. If technological progress is more rapid when there are more people to discover things, then the more populous regions should have experienced more rapid growth.

And, indeed, they did. The most successful region of the world in 1500 (when Columbus reestablished technological contact) included the "Old World" civilizations of the large Eurasia–Africa region. Next in technological development were the Aztec and Mayan civilizations in the Americas, followed by the hunter-gatherers of Australia, and then the primitive people of Tasmania, who lacked even fire-making and most stone and bone tools. The least populous isolated region was Flinders Island, a tiny island between Tasmania and Australia. With

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[6]For modern analyses of the Malthusian model, see Oded Galor and David N. Weil, "Population, Technology, and Growth: From Malthusian Stagnation to the Demographic Transition and Beyond," *American Economic Review* 90 (September 2000): 806–828; and Gary D. Hansen and Edward C. Prescott, "Malthus to Solow," *American Economic Review* 92 (September 2002): 1205–1217.

few people to contribute new innovations, Flinders Island had the least technological advance and, in fact, seemed to regress. Around 3000 B.C., human society on Flinders Island died out completely.

Kremer concludes from this evidence that a large population is a prerequisite for technological advance.[7]

8-4 Conclusion

This chapter has started the process of building the Solow growth model. The model as developed so far shows how saving and population growth determine the economy's steady-state capital stock and its steady-state level of income per person. As we have seen, it sheds light on many features of actual growth experiences—why Germany and Japan grew so rapidly after being devastated by World War II, why countries that save and invest a high fraction of their output are richer than countries that save and invest a smaller fraction, and why countries with high rates of population growth are poorer than countries with low rates of population growth.

What the model cannot do, however, is explain the persistent growth in living standards we observe in most countries. In the model we have developed so far, output per worker stops growing when the economy reaches its steady state. To explain persistent growth, we need to introduce technological progress into the model. That is our first job in the next chapter.

Summary

1. The Solow growth model shows that in the long run, an economy's rate of saving determines the size of its capital stock and thus its level of production. The higher the rate of saving, the higher the stock of capital and the higher the level of output.
2. In the Solow model, an increase in the rate of saving has a level effect on income per person: it causes a period of rapid growth, but eventually that growth slows as the new steady state is reached. Thus, although a high saving rate yields a high steady-state level of output, saving by itself cannot generate persistent economic growth.
3. The level of capital that maximizes steady-state consumption is called the Golden Rule level. If an economy has more capital than in the Golden Rule steady state, then reducing saving will increase consumption at all points in time. By contrast, if the economy has less capital than in the Golden Rule steady state, then reaching the Golden Rule requires increased investment and thus lower consumption for current generations.

[7]Michael Kremer, "Population Growth and Technological Change: One Million B.C. to 1990," *Quarterly Journal of Economics* 108 (August 1993): 681–716.

4. The Solow model shows that an economy's rate of population growth is another long-run determinant of the standard of living. According to the Solow model, the higher the rate of population growth, the lower the steady-state levels of capital per worker and output per worker. Other theories highlight other effects of population growth. Malthus suggested that population growth will strain the natural resources necessary to produce food; Kremer suggested that a large population may promote technological progress.

KEY CONCEPTS

Solow growth model

Steady state

Golden Rule level of capital

QUESTIONS FOR REVIEW

1. In the Solow model, how does the saving rate affect the steady-state level of income? How does it affect the steady-state rate of growth?
2. Why might an economic policymaker choose the Golden Rule level of capital?
3. Might a policymaker choose a steady state with more capital than in the Golden Rule steady state? With less capital than in the Golden Rule steady state? Explain your answers.
4. In the Solow model, how does the rate of population growth affect the steady-state level of income? How does it affect the steady-state rate of growth?

PROBLEMS AND APPLICATIONS

1. Country A and country B both have the production function

$$Y = F(K, L) = K^{1/2}L^{1/2}.$$

   a. Does this production function have constant returns to scale? Explain.

   b. What is the per-worker production function, $y = f(k)$?

   c. Assume that neither country experiences population growth or technological progress and that 5 percent of capital depreciates each year. Assume further that country A saves 10 percent of output each year and country B saves 20 percent of output each year. Using your answer from part (b) and the steady-state condition that investment equals depreciation, find the steady-state level of capital per worker for each country. Then find the steady-state levels of income per worker and consumption per worker.

   d. Suppose that both countries start off with a capital stock per worker of 2. What are the levels of income per worker and consumption per worker? Remembering that the change in the capital stock is investment less depreciation, use a calculator or a computer spreadsheet to show how the capital stock per worker will evolve over time in both countries. For each year, calculate income per worker and consumption per worker. How many years will it be before the consumption in country B is higher than the consumption in country A?

2. In the discussion of German and Japanese postwar growth, the text describes what happens when part of the capital stock is destroyed in a war. By contrast, suppose that a war does not directly affect the capital stock, but that casualties reduce the labor force. Assume the economy was in a steady state before the war, the saving rate

is unchanged, and the rate of population growth after the war is the same as it was before.

a. What is the immediate impact of the war on total output and on output per person?

b. What happens subsequently to output per worker in the postwar economy? Is the growth rate of output per worker after the war smaller or greater than it was before the war?

3. Consider an economy described by the production function: $Y = F(K, L) = K^{0.3}L^{0.7}$.

a. What is the per-worker production function?

b. Assuming no population growth or technological progress, find the steady-state capital stock per worker, output per worker, and consumption per worker as a function of the saving rate and the depreciation rate.

c. Assume that the depreciation rate is 10 percent per year. Make a table showing steady-state capital per worker, output per worker, and consumption per worker for saving rates of 0 percent, 10 percent, 20 percent, 30 percent, and so on. (You will need a calculator with an exponent key for this.) What saving rate maximizes output per worker? What saving rate maximizes consumption per worker?

d. (Harder) Use calculus to find the marginal product of capital. Add to your table from part (c) the marginal product of capital net of depreciation for each of the saving rates. What does your table show about the relationship between the net marginal product of capital and steady-state consumption?

4. "Devoting a larger share of national output to investment would help restore rapid productivity growth and rising living standards." Do you agree with this claim? Explain, using the Solow model.

5. Draw a well-labeled graph that illustrates the steady state of the Solow model with population growth. Use the graph to find what happens to steady-state capital per worker and income per worker in response to each of the following exogenous changes.

a. A change in consumer preferences increases the saving rate.

b. A change in weather patterns increases the depreciation rate.

c. Better birth-control methods reduce the rate of population growth.

d. A one-time, permanent improvement in technology increases the amount of output that can be produced from any given amount of capital and labor.

6. Many demographers predict that the United States will have zero population growth in the twenty-first century, in contrast to average population growth of about 1 percent per year in the twentieth century. Use the Solow model to forecast the effect of this slowdown in population growth on the growth of total output and the growth of output per person. Consider the effects both in the steady state and in the transition between steady states.

7. In the Solow model, population growth leads to steady-state growth in total output, but not in output per worker. Do you think this would still be true if the production function exhibited increasing or decreasing returns to scale? Explain. (For the definitions of increasing and decreasing returns to scale, see Chapter 3, "Problems and Applications," Problem 3.)

8. Consider how unemployment would affect the Solow growth model. Suppose that output is produced according to the production function $Y = K^{\alpha}[(1 - u)L]^{1-\alpha}$, where K is capital, L is the labor force, and u is the natural rate of unemployment. The national saving rate is s, the labor force grows at rate n, and capital depreciates at rate δ.

a. Express output per worker ($y = Y/L$) as a function of capital per worker ($k = K/L$) and the natural rate of unemployment (u).

b. Write an equation that describes the steady state of this economy. Illustrate the steady state graphically, as we did in this chapter for the standard Solow model.

c. Suppose that some change in government policy reduces the natural rate of unemployment. Using the graph you drew in part (b), describe how this change affects output both immediately and over time. Is the steady-state effect on output larger or smaller than the immediate effect? Explain.

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CHAPTER 9

Economic Growth II: Technology, Empirics, and Policy

> *Is there some action a government of India could take that would lead the Indian economy to grow like Indonesia's or Egypt's? If so, what, exactly? If not, what is it about the "nature of India" that makes it so? The consequences for human welfare involved in questions like these are simply staggering: Once one starts to think about them, it is hard to think about anything else.*
>
> —*Robert E. Lucas, Jr.*

The quotation that opens this chapter was written in 1988. Since then, India has grown rapidly, a phenomenon that has pulled millions of people out of extreme poverty. At the same time, some other poor nations, including many in sub-Saharan Africa, have experienced little growth, and their citizens continue to live meager existences. It is the job of growth theory to explain such disparate outcomes. The reasons why some nations succeed while others fail at promoting long-run economic growth are not easily apparent, but as Robert Lucas suggests, the consequences for human welfare are indeed staggering.

This chapter continues our analysis of the forces governing long-run growth. With the basic version of the Solow model as our starting point, we take on four new tasks.

Our first task is to make the Solow model more general and realistic. In Chapter 3 we saw that capital, labor, and technology are the key determinants of a nation's production of goods and services. In Chapter 8 we developed the Solow model to show how changes in capital (through saving and investment) and changes in the labor force (through population growth) affect the economy's output. We are now ready to add the third source of growth—changes in technology—to the mix. The Solow model does not explain technological progress but, instead, takes it as exogenously given and shows how it interacts with other variables in the process of economic growth.

Our second task is to move from theory to empirics. That is, we consider how well the Solow model fits the facts. Over the past two decades, a large literature has examined the predictions of the Solow model and other models of economic growth. It turns out that the glass is both half full and half empty. The Solow model can shed much light on international growth experiences, but it is far from the last word on the subject.

Our third task is to examine how a nation's public policies can influence the level and growth of its citizens' standard of living. In particular, we address five questions: Should our society save more or less? How can policy influence the rate of saving? Are there some types of investment that policy should especially encourage? What institutions ensure that the economy's resources are put to their best use? How can policy increase the rate of technological progress? The Solow growth model provides the theoretical framework within which we consider these policy issues.

Our fourth and final task is to consider what the Solow model leaves out. As we have discussed previously, models help us understand the world by simplifying it. After completing an analysis of a model, therefore, it is important to consider whether we have oversimplified matters. In the last section, we examine a new set of theories, called *endogenous growth theories*, which help to explain the technological progress that the Solow model takes as exogenous.

9-1 Technological Progress in the Solow Model

So far, our presentation of the Solow model has assumed an unchanging relationship between the inputs of capital and labor and the output of goods and services. Yet the model can be modified to include exogenous technological progress, which over time expands society's production capabilities.

The Efficiency of Labor

To incorporate technological progress, we must return to the production function that relates total capital K and total labor L to total output Y. Thus far, the production function has been

$$Y = F(K, L).$$

We now write the production function as

$$Y = F(K, L \times E),$$

where E is a new (and somewhat abstract) variable called the **efficiency of labor**. The efficiency of labor is meant to reflect society's knowledge about production methods: as the available technology improves, the efficiency of labor rises, and each hour of work contributes more to the production of goods and services. For instance, the efficiency of labor rose when assembly-line production transformed manufacturing in the early twentieth century, and it rose again when computerization was introduced in the late twentieth century. The efficiency of labor also rises when there are improvements in the health, education, or skills of the labor force.

The term $L \times E$ can be interpreted as measuring the *effective number of workers*. It takes into account the number of actual workers L and the efficiency of each worker E. In other words, L measures the number of workers in the labor force, whereas $L \times E$ measures both the workers and the technology with which the typical worker comes equipped. This new production function states that total output Y depends on the inputs of capital K and effective workers $L \times E$.

The essence of this approach to modeling technological progress is that increases in the efficiency of labor E are analogous to increases in the labor force L. Suppose, for example, that an advance in production methods makes the efficiency of labor E double between 1980 and 2012. This means that a single worker in 2012 is, *in effect*, as productive as two workers were in 1980. That is, even if the actual number of workers (L) stays the same from 1980 to 2012, the effective number of workers ($L \times E$) doubles, and the economy benefits from the increased production of goods and services.

The simplest assumption about technological progress is that it causes the efficiency of labor E to grow at some constant rate g. For example, if $g = 0.02$, then each unit of labor becomes 2 percent more efficient each year: output increases as if the labor force had increased by 2 percent more than it really did. This form of technological progress is called *labor augmenting*, and g is called the rate of **labor-augmenting technological progress**. Because the labor force L is growing at rate n, and the efficiency of each unit of labor E is growing at rate g, the effective number of workers $L \times E$ is growing at rate $n + g$.

The Steady State With Technological Progress

Because technological progress is modeled here as labor augmenting, it fits into the model in much the same way as population growth. Technological progress does not cause the actual number of workers to increase, but because each worker in effect comes with more units of labor over time, technological progress causes the effective number of workers to increase. Thus, the analytic tools we used in Chapter 8 to study the Solow model with population growth are easily adapted to studying the Solow model with labor-augmenting technological progress.

We begin by reconsidering our notation. Previously, when there was no technological progress, we analyzed the economy in terms of quantities per worker; now we can generalize that approach by analyzing the economy in terms of quantities per effective worker. We now let $k = K/(L \times E)$ stand for capital per effective worker and $y = Y/(L \times E)$ stand for output per effective worker. With these definitions, we can again write $y = f(k)$.

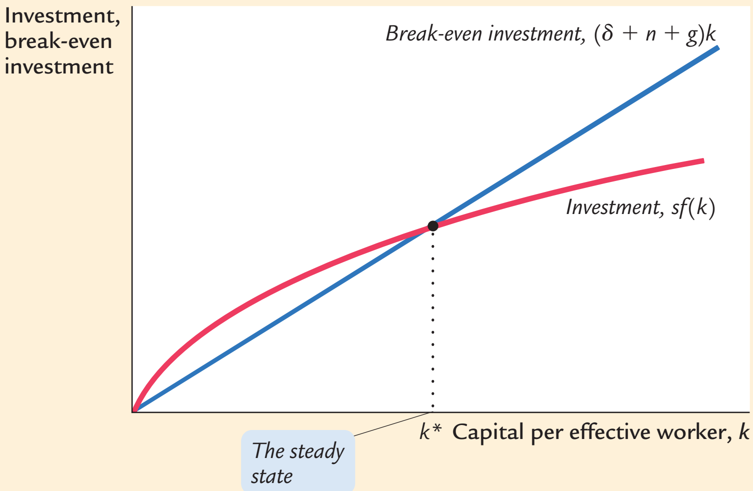
Our analysis of the economy proceeds just as it did when we examined population growth. The equation showing the evolution of k over time becomes

$$\Delta k = sf(k) - (\delta + n + g)k.$$

As before, the change in the capital stock Δk equals investment $sf(k)$ minus break-even investment $(\delta + n + g)k$. Now, however, because $k = K/(L \times E)$, break-even investment includes three terms: to keep k constant, δk is needed to replace depreciating capital, nk is needed to provide capital for new workers, and gk is needed to provide capital for the new "effective workers" created by technological progress.[1]

[1]*Mathematical note:* This model with technological progress is a strict generalization of the model analyzed in Chapter 8. In particular, if the efficiency of labor is constant at $E = 1$, then $g = 0$, and the definitions of k and y reduce to our previous definitions. In this case, the more general model considered here simplifies precisely to the Chapter 8 version of the Solow model.

FIGURE 9-1



Technological Progress and the Solow Growth Model Labor-augmenting technological progress at rate g enters our analysis of the Solow growth model in much the same way as did population growth at rate n. Now that k is defined as the amount of capital per effective worker, increases in the effective number of workers because of technological progress tend to decrease k. In the steady state, investment $sf(k)$ exactly offsets the reductions in k attributable to depreciation, population growth, and technological progress.

As shown in Figure 9-1, the inclusion of technological progress does not substantially alter our analysis of the steady state. There is one level of k, denoted k^*, at which capital per effective worker and output per effective worker are constant. As before, this steady state represents the long-run equilibrium of the economy.

The Effects of Technological Progress

Table 9-1 shows how four key variables behave in the steady state with technological progress. As we have just seen, capital per effective worker k is constant in the steady state. Because $y = f(k)$, output per effective worker is also constant. It is these quantities per effective worker that are steady in the steady state.

From this information, we can also infer what is happening to variables that are not expressed in units per effective worker. For instance, consider output per actual

TABLE 9-1

Steady-State Growth Rates in the Solow Model With Technological Progress

Variable	Symbol	Steady-State Growth Rate
Capital per effective worker	$k = K/(E \times L)$	0
Output per effective worker	$y = Y/(E \times L) = f(k)$	0
Output per worker	$Y/L = y \times E$	g
Total output	$Y = y \times (E \times L)$	$n + g$

worker $Y/L = y \times E$. Because y is constant in the steady state and E is growing at rate g, output per worker must also be growing at rate g in the steady state. Similarly, the economy's total output is $Y = y \times (E \times L)$. Because y is constant in the steady state, E is growing at rate g, and L is growing at rate n, total output grows at rate $n + g$ in the steady state.

With the addition of technological progress, our model can finally explain the sustained increases in standards of living that we observe. That is, we have shown that technological progress can lead to sustained growth in output per worker. By contrast, a high rate of saving leads to a high rate of growth only until the steady state is reached. Once the economy is in steady state, the rate of growth of output per worker depends only on the rate of technological progress. *According to the Solow model, only technological progress can explain sustained growth and persistently rising living standards.*

The introduction of technological progress also modifies the criterion for the Golden Rule. The Golden Rule level of capital is now defined as the steady state that maximizes consumption per effective worker. Following the same arguments that we have used before, we can show that steady-state consumption per effective worker is

$$c^* = f(k^*) - (\delta + n + g)k^*.$$

Steady-state consumption is maximized if

$$MPK = \delta + n + g,$$

or

$$MPK - \delta = n + g.$$

That is, at the Golden Rule level of capital, the net marginal product of capital, $MPK - \delta$, equals the rate of growth of total output, $n + g$. Because actual economies experience both population growth and technological progress, we must use this criterion to evaluate whether they have more or less capital than they would at the Golden Rule steady state.

9-2 From Growth Theory to Growth Empirics

So far in this chapter we have introduced exogenous technological progress into the Solow model to explain sustained growth in standards of living. Let's now discuss what happens when this theory is forced to confront the facts.

Balanced Growth

According to the Solow model, technological progress causes the values of many variables to rise together in the steady state. This property, called *balanced growth*, does a good job of describing the long-run data for the U.S. economy.

Consider first output per worker Y/L and the capital stock per worker K/L. According to the Solow model, in the steady state both of these variables grow at g, the rate of technological progress. U.S. data for the past half century show that output per worker and the capital stock per worker have in fact grown at approximately the same rate—about 2 percent per year. To put it another way, the capital–output ratio has remained approximately constant over time.

Technological progress also affects factor prices. Problem 3(d) at the end of the chapter asks you to show that, in the steady state, the real wage grows at the rate of technological progress. The real rental price of capital, however, is constant over time. Again, these predictions hold true for the United States. Over the past 50 years, the real wage has increased about 2 percent per year; it has increased at about the same rate as real GDP per worker. Yet the real rental price of capital (measured as real capital income divided by the capital stock) has remained about the same.

The Solow model's prediction about factor prices—and the success of this prediction—is especially noteworthy when contrasted with Karl Marx's theory of the development of capitalist economies. Marx predicted that the return to capital would decline over time and that this would lead to economic and political crisis. Economic history has not supported Marx's prediction, which partly explains why we now study Solow's theory of growth rather than Marx's.

Convergence

If you travel around the world, you will see tremendous variation in living standards. The world's poor countries have average levels of income per person that are less than one-tenth the average levels in the world's rich countries. These differences in income are reflected in almost every measure of the quality of life—from the number of televisions and telephones per household to the infant mortality rate and life expectancy.

Much research has been devoted to the question of whether economies converge over time to one another. In particular, do economies that start off poor subsequently grow faster than economies that start off rich? If they do, then the world's poor economies will tend to catch up with the world's rich economies. This process of catch-up is called *convergence*. If convergence does not occur, then countries that start off behind are likely to remain poor.

The Solow model makes clear predictions about when convergence should occur. According to the model, whether two economies will converge depends on why they differ in the first place. On the one hand, suppose two economies happen by historical accident to start off with different capital stocks, but they have the same steady state, as determined by their saving rates, population growth rates, and efficiency of labor. In this case, we should expect the two economies to converge; the poorer economy with the smaller capital stock will naturally grow more quickly to reach the steady state. (In a Case Study in Chapter 8, we applied this logic to explain rapid growth in Germany and Japan after World War II.) On the other hand, if two economies have different steady states, perhaps because the economies have different rates of saving, then we should not expect convergence. Instead, each economy will approach its own steady state.

Experience is consistent with this analysis. In samples of economies with similar cultures and policies, studies find that economies converge to one another at a rate of about 2 percent per year. That is, the gap between rich and poor economies closes by about 2 percent each year. An example is the economies of individual American states. For historical reasons, such as the Civil War of the 1860s, income levels varied greatly among states at the end of the nineteenth century. Yet these differences have slowly disappeared over time.

In international data, a more complex picture emerges. When researchers examine only data on income per person, they find little evidence of convergence: countries that start off poor do not grow faster on average than countries that start off rich. This finding suggests that different countries have different steady states. If statistical techniques are used to control for some of the determinants of the steady state, such as saving rates, population growth rates, and accumulation of human capital (education), then once again the data show convergence at a rate of about 2 percent per year. In other words, the economies of the world exhibit *conditional convergence*: they appear to be converging to their own steady states, which in turn are determined by such variables as saving, population growth, and human capital.[2]

Factor Accumulation Versus Production Efficiency

As a matter of accounting, international differences in income per person can be attributed to either (1) differences in the factors of production, such as the quantities of physical and human capital, or (2) differences in the efficiency with which economies use their factors of production. That is, a worker in a poor country may be poor because he lacks tools and skills or because the tools and skills he has are not being put to their best use. To describe this issue in terms of the Solow model, the question is whether the large gap between rich and poor is explained by differences in capital accumulation (including human capital) or differences in the production function.

Much research has attempted to estimate the relative importance of these two sources of income disparities. The exact answer varies from study to study, but both factor accumulation and production efficiency appear important. Moreover, a common finding is that they are positively correlated: nations with high levels of physical and human capital also tend to use those factors efficiently.[3]

[2]Robert Barro and Xavier Sala-i-Martin, "Convergence Across States and Regions," *Brookings Papers on Economic Activity* 1 (1991): 107–182; N. Gregory Mankiw, David Romer, and David N. Weil, "A Contribution to the Empirics of Economic Growth," *Quarterly Journal of Economics* (May 1992): 407–437.

[3]Robert E. Hall and Charles I. Jones, "Why Do Some Countries Produce So Much More Output per Worker Than Others?" *Quarterly Journal of Economics* 114 (February 1999): 83–116; Peter J. Klenow and Andres Rodriguez-Clare, "The Neoclassical Revival in Growth Economics: Has It Gone Too Far?" *NBER Macroeconomics Annual* (1997): 73–103.

There are several ways to interpret this positive correlation. One hypothesis is that an efficient economy may encourage capital accumulation. For example, a person in a well-functioning economy may have greater resources and incentive to stay in school and accumulate human capital. Another hypothesis is that capital accumulation may induce greater efficiency. If there are positive externalities to physical and human capital, then countries that save and invest more will appear to have better production functions (unless the research study accounts for these externalities, which is hard to do). Thus, greater production efficiency may cause greater factor accumulation, or the other way around.

A final hypothesis is that both factor accumulation and production efficiency are driven by a common third variable. Perhaps the common third variable is the quality of the nation's institutions, including the government's policymaking process. As one economist put it, when governments screw up, they screw up big time. Bad policies, such as high inflation, excessive budget deficits, widespread market interference, and rampant corruption, often go hand in hand. We should not be surprised that economies exhibiting these maladies both accumulate less capital and fail to use the capital they have as efficiently as they might.

CASE STUDY

Is Free Trade Good for Economic Growth?

At least since Adam Smith, economists have advocated free trade as a policy that promotes national prosperity. Here is how Smith put the argument in his 1776 classic, *The Wealth of Nations:*

> It is a maxim of every prudent master of a family, never to attempt to make at home what it will cost him more to make than to buy. The tailor does not attempt to make his own shoes, but buys them of the shoemaker. The shoemaker does not attempt to make his own clothes but employs a tailor. . . .
>
> What is prudence in the conduct of every private family can scarce be folly in that of a great kingdom. If a foreign country can supply us with a commodity cheaper than we ourselves can make it, better buy it of them with some part of the produce of our own industry employed in a way in which we have some advantage.

Today, economists make the case with greater rigor, relying on David Ricardo's theory of comparative advantage as well as more modern theories of international trade. According to these theories, a nation open to trade can achieve greater production efficiency and a higher standard of living by specializing in those goods for which it has a comparative advantage.

A skeptic might point out that this is just a theory. What about the evidence? Do nations that permit free trade in fact enjoy greater prosperity? A large body of literature addresses precisely this question.

One approach is to look at international data to see if countries that are open to trade typically enjoy greater prosperity. The evidence shows that they do. Economists Andrew Warner and Jeffrey Sachs studied this question for the period from 1970 to 1989. They report that among developed nations, the open economies grew at 2.3 percent per year, while the closed economies grew at 0.7 percent per year. Among developing nations, the open economies grew at 4.5 percent per year, while the closed economies again grew at 0.7 percent per year. These findings are consistent with Smith's view that trade enhances prosperity, but they are not conclusive. Correlation does not prove causation. Perhaps being closed to trade is correlated with various other restrictive government policies, and it is those other policies that retard growth.

A second approach is to look at what happens when closed economies remove their trade restrictions. Once again, Smith's hypothesis fares well. Throughout history, when nations open themselves up to the world economy, the typical result is a subsequent increase in economic growth. This occurred in Japan in the 1850s, South Korea in the 1960s, and Vietnam in the 1990s. But once again, correlation does not prove causation. Trade liberalization is often accompanied by other reforms, and it is hard to disentangle the effects of trade from the effects of the other reforms.

A third approach to measuring the impact of trade on growth, proposed by economists Jeffrey Frankel and David Romer, is to look at the impact of geography. Some countries trade less simply because they are geographically disadvantaged. For example, New Zealand is disadvantaged compared to Belgium because it is farther from other populous countries. Similarly, landlocked countries are disadvantaged compared to countries with their own seaports. Because these geographical characteristics are correlated with trade, but arguably uncorrelated with other determinants of economic prosperity, they can be used to identify the causal impact of trade on income. (The statistical technique, which you may have studied in an econometrics course, is called *instrumental variables*.) After analyzing the data, Frankel and Romer conclude that "a rise of one percentage point in the ratio of trade to GDP increases income per person by at least one-half percentage point. Trade appears to raise income by spurring the accumulation of human and physical capital and by increasing output for given levels of capital."

The overwhelming weight of the evidence from this body of research is that Adam Smith was right. Openness to international trade is good for economic growth.[4] ■

9-3 Policies to Promote Growth

So far we have used the Solow model to uncover the theoretical relationships among the different sources of economic growth, and we have discussed some of the empirical work that describes actual growth experiences. We can now use the theory and evidence to help guide our thinking about economic policy.

[4]Jeffrey D. Sachs and Andrew Warner, "Economic Reform and the Process of Global Integration," *Brookings Papers on Economic Activity* (1995): 1–95; Jeffrey A. Frankel and David Romer, "Does Trade Cause Growth?" *American Economics Review* 89 (June 1999): 379–399.

Evaluating the Rate of Saving

According to the Solow growth model, how much a nation saves and invests is a key determinant of its citizens' standard of living. So let's begin our policy discussion with a natural question: is the rate of saving in the U.S. economy too low, too high, or about right?

As we have seen, the saving rate determines the steady-state levels of capital and output. One particular saving rate produces the Golden Rule steady state, which maximizes consumption per worker and thus economic well-being. The Golden Rule provides the benchmark against which we can compare the U.S. economy.

To decide whether the U.S. economy is at, above, or below the Golden Rule steady state, we need to compare the marginal product of capital net of depreciation ($MPK - \delta$) with the growth rate of total output ($n + g$). As we established in Section 9-1, at the Golden Rule steady state, $MPK - \delta = n + g$. If the economy is operating with less capital than in the Golden Rule steady state, then diminishing marginal product tells us that $MPK - \delta > n + g$. In this case, increasing the rate of saving will increase capital accumulation and economic growth and, eventually, lead to a steady state with higher consumption (although consumption will be lower for part of the transition to the new steady state). On the other hand, if the economy has more capital than in the Golden Rule steady state, then $MPK - \delta < n + g$. In this case, capital accumulation is excessive: reducing the rate of saving will lead to higher consumption both immediately and in the long run.

To make this comparison for a real economy, such as the U.S. economy, we need an estimate of the growth rate of output ($n + g$) and an estimate of the net marginal product of capital ($MPK - \delta$). Real GDP in the United States grows an average of 3 percent per year, so $n + g = 0.03$. We can estimate the net marginal product of capital from the following three facts:

1. The capital stock is about 2.5 times one year's GDP.
2. Depreciation of capital is about 10 percent of GDP.
3. Capital income is about 30 percent of GDP.

Using the notation of our model (and the result from Chapter 3 that capital owners earn income of MPK for each unit of capital), we can write these facts as

1. $k = 2.5y$.
2. $\delta k = 0.1y$.
3. $MPK \times k = 0.3y$.

We solve for the rate of depreciation δ by dividing equation 2 by equation 1:

$$\delta k/k = (0.1y)/(2.5y)$$
$$\delta = 0.04.$$

And we solve for the marginal product of capital MPK by dividing equation 3 by equation 1:

$$(MPK \times k)/k = (0.3y)/(2.5y)$$
$$MPK = 0.12.$$

Thus, about 4 percent of the capital stock depreciates each year, and the marginal product of capital is about 12 percent per year. The net marginal product of capital, $MPK - \delta$, is about 8 percent per year.

We can now see that the return to capital ($MPK - \delta = 8$ percent per year) is well in excess of the economy's average growth rate ($n + g = 3$ percent per year). This fact, together with our previous analysis, indicates that the capital stock in the U.S. economy is well below the Golden Rule level. In other words, if the United States saved and invested a higher fraction of its income, it would grow more rapidly and eventually reach a steady state with higher consumption.

This conclusion is not unique to the U.S. economy. When calculations similar to those above are done for other economies, the results are similar. The possibility of excessive saving and capital accumulation beyond the Golden Rule level is intriguing as a matter of theory, but it appears not to be a problem that actual economies face. In practice, economists are more often concerned with insufficient saving. It is this kind of calculation that provides the intellectual foundation for this concern.[5]

Changing the Rate of Saving

The preceding calculations show that to move the U.S. economy toward the Golden Rule steady state, policymakers should increase national saving. But how can they do that? We saw in Chapter 3 that, as a matter of sheer accounting, higher national saving means higher public saving, higher private saving, or some combination of the two. Much of the debate over policies to increase growth centers on which of these options is likely to be most effective.

The most direct way in which the government affects national saving is through public saving—the difference between what the government receives in tax revenue and what it spends. When its spending exceeds its revenue, the government runs a *budget deficit*, which represents negative public saving. As we saw in Chapter 3, a budget deficit raises interest rates and crowds out investment; the resulting reduction in the capital stock is part of the burden of the national debt on future generations. Conversely, if it spends less than it raises in revenue, the government runs a *budget surplus*, which it can use to retire some of the national debt and stimulate investment.

The government also affects national saving by influencing private saving—the saving done by households and firms. In particular, how much people decide to save depends on the incentives they face, and these incentives are altered by a variety of public policies. Many economists argue that high tax rates on capital—including the corporate income tax, the federal income tax, the estate tax, and many state income and estate taxes—discourage private saving by reducing the rate of return that savers earn. On the other hand, tax-exempt retirement accounts, such as IRAs, are designed to encourage private saving by giving preferential treatment to income saved in these accounts. Some economists have proposed increasing the incentive to save by replacing the current system of income taxation with a system of consumption taxation.

[5]For more on this topic and some international evidence, see Andrew B. Abel, N. Gregory Mankiw, Lawrence H. Summers, and Richard J. Zeckhauser, "Assessing Dynamic Efficiency: Theory and Evidence," *Review of Economic Studies* 56 (1989): 1–19.

Many disagreements over public policy are rooted in different views about how much private saving responds to incentives. For example, suppose that the government increased the amount that people could put into tax-exempt retirement accounts. Would people respond to this incentive by saving more? Or, instead, would people merely transfer saving already done in other forms into these accounts—reducing tax revenue and thus public saving without any stimulus to private saving? The desirability of the policy depends on the answers to these questions. Unfortunately, despite much research on this issue, no consensus has emerged.

Allocating the Economy's Investment

The Solow model makes the simplifying assumption that there is only one type of capital. In the world, of course, there are many types. Private businesses invest in traditional types of capital, such as bulldozers and steel plants, and newer types of capital, such as computers and robots. The government invests in various forms of public capital, called *infrastructure*, such as roads, bridges, and sewer systems.

In addition, there is *human capital*—the knowledge and skills that workers acquire through education, from early-childhood programs such as Head Start to on-the-job training for adults in the labor force. Although the capital variable in the Solow model is usually interpreted as including only physical capital, in many ways human capital is analogous to physical capital. Like physical capital, human capital increases our ability to produce goods and services. Raising the level of human capital requires investment in the form of teachers, libraries, and student time. Research on economic growth has emphasized that human capital is at least as important as physical capital in explaining international differences in standards of living. One way of modeling this fact is to give the variable we call "capital" a broader definition that includes both human and physical capital.[6]

Policymakers trying to promote economic growth must confront the issue of what kinds of capital the economy needs most. In other words, what kinds of capital yield the highest marginal products? To a large extent, policymakers can rely on the marketplace to allocate the pool of saving to alternative types of investment. Those industries with the highest marginal products of capital will naturally be most willing to borrow at market interest rates to finance new investment. Many economists advocate that the government should merely create a "level playing field" for different types of capital—for example, by ensuring that the tax system treats all forms of capital equally. The government can then rely on the market to allocate capital efficiently.

Other economists have suggested that the government should actively encourage particular forms of capital. Suppose, for instance, that technological advance

[6]Earlier in this chapter, when we were interpreting K as only physical capital, human capital was folded into the efficiency-of-labor parameter E. The alternative approach suggested here is to include human capital as part of K instead, so E represents technology but not human capital. If K is given this broader interpretation, then much of what we call labor income is really the return to human capital. As a result, the true capital share is much larger than the traditional Cobb–Douglas value of about 1/3. For more on this topic, see N. Gregory Mankiw, David Romer, and David N. Weil, "A Contribution to the Empirics of Economic Growth," *Quarterly Journal of Economics* (May 1992): 407–437.

occurs as a by-product of certain economic activities. This would happen if new and improved production processes are devised during the process of building capital (a phenomenon called *learning by doing*) and if these ideas become part of society's pool of knowledge. Such a by-product is called a *technological externality* (or a *knowledge spillover*). In the presence of such externalities, the social returns to capital exceed the private returns, and the benefits of increased capital accumulation to society are greater than the Solow model suggests.[7] Moreover, some types of capital accumulation may yield greater externalities than others. If, for example, installing robots yields greater technological externalities than building a new steel mill, then perhaps the government should use the tax laws to encourage investment in robots. The success of such an *industrial policy*, as it is sometimes called, requires that the government be able to accurately measure the externalities of different economic activities so it can give the correct incentive to each activity.

Most economists are skeptical about industrial policies for two reasons. First, measuring the externalities from different sectors is virtually impossible. If policy is based on poor measurements, its effects might be close to random and, thus, worse than no policy at all. Second, the political process is far from perfect. Once the government gets into the business of rewarding specific industries with subsidies and tax breaks, the rewards are as likely to be based on political clout as on the magnitude of externalities.

One type of capital that necessarily involves the government is public capital. Local, state, and federal governments are always deciding if and when they should borrow to finance new roads, bridges, and transit systems. In 2009, one of President Barack Obama's first economic proposals was to increase spending on such infrastructure. This policy was motivated by a desire partly to increase short-run aggregate demand (a goal we will examine later in this book) and partly to provide public capital and enhance long-run economic growth. Among economists, this policy had both defenders and critics. Yet all of them agree that measuring the marginal product of public capital is difficult. Private capital generates an easily measured rate of profit for the firm owning the capital, whereas the benefits of public capital are more diffuse. Furthermore, while private capital investment is made by investors spending their own money, the allocation of resources for public capital involves the political process and taxpayer funding. It is all too common to see "bridges to nowhere" being built simply because the local senator or congressman has the political muscle to get funds approved.

CASE STUDY

Industrial Policy in Practice

Policymakers and economists have long debated whether the government should promote certain industries and firms because they are strategically important for the economy. In the United States, the debate goes back over two centuries.

[7]Paul Romer, "Crazy Explanations for the Productivity Slowdown," *NBER Macroeconomics Annual* 2 (1987): 163–201.

Alexander Hamilton, the first U.S. Secretary of the Treasury, favored tariffs on certain imports to encourage the development of domestic manufacturing. The Tariff of 1789 was the second act passed by the new federal government. The tariff helped manufacturers, but it hurt farmers, who had to pay more for foreign-made products. Because the North was home to most of the manufacturers, while the South had more farmers, the tariff was one source of the regional tensions that eventually led to the Civil War.

Advocates of a significant government role in promoting technology can point to some recent successes. For example, the precursor of the modern Internet is a system called Arpanet, which was established by an arm of the U.S. Department of Defense as a way for information to flow among military installations. There is little doubt that the Internet has been associated with large advances in productivity and that the government had a hand in its creation. According to proponents of industrial policy, this example illustrates how the government can help jump-start an emerging technology.

Yet governments can also make mistakes when they try to supplant private business decisions. Japan's Ministry of International Trade and Industry (MITI) is sometimes viewed as a successful practitioner of industrial policy, but it once tried to stop Honda from expanding its business from motorcycles to automobiles. MITI thought that the nation already had enough car manufacturers. Fortunately, the government lost this battle, and Honda turned into one of the world's largest and most profitable car companies. Soichiro Honda, the company's founder, once said, "Probably I would have been even more successful had we not had MITI."

Over the past several years, government policy has aimed to promote "green technologies." In particular, the U.S. federal government has subsidized the production of energy in ways that yield lower carbon emissions, which are thought to contribute to global climate change. It is too early to judge the long-run success of this policy, but there have been some short-run embarrassments. In 2011, a manufacturer of solar panels called Solyndra declared bankruptcy two years after the federal government granted it a $535 million loan guarantee. Moreover, there were allegations that the decision to grant the loan guarantee had been politically motivated rather than based on an objective evaluation of Solyndra's business plan. As this book was going to press, the Solyndra case was under investigation by congressional committees and the FBI.

The debate over industrial policy will surely continue in the years to come. The final judgment about this kind of government intervention in the market requires evaluating both the efficiency of unfettered markets and the ability of governmental institutions to identify technologies worthy of support. ■

Establishing the Right Institutions

As we discussed earlier, economists who study international differences in the standard of living attribute some of these differences to the inputs of physical and human capital and some to the productivity with which these inputs are used. One reason nations may have different levels of production efficiency is that they have different institutions guiding the allocation of scarce resources. Creating the right institutions is important for ensuring that resources are allocated to their best use.

A nation's legal tradition is an example of such an institution. Some countries, such as the United States, Australia, India, and Singapore, are former colonies of the United Kingdom and, therefore, have English-style common-law systems. Other nations, such as Italy, Spain, and most of those in Latin America, have legal traditions that evolved from the French Napoleonic Code. Studies have found that legal protections for shareholders and creditors are stronger in English-style than French-style legal systems. As a result, the English-style countries have better-developed capital markets. Nations with better-developed capital markets, in turn, experience more rapid growth because it is easier for small and start-up companies to finance investment projects, leading to a more efficient allocation of the nation's capital.[8]

Another important institutional difference across countries is the quality of government itself. Ideally, governments should provide a "helping hand" to the market system by protecting property rights, enforcing contracts, promoting competition, prosecuting fraud, and so on. Yet governments sometimes diverge from this ideal and act more like a "grabbing hand" by using the authority of the state to enrich a few powerful individuals at the expense of the broader community. Empirical studies have shown that the extent of corruption in a nation is indeed a significant determinant of economic growth.[9]

Adam Smith, the great eighteenth-century economist, was well aware of the role of institutions in economic growth. He once wrote, "Little else is requisite to carry a state to the highest degree of opulence from the lowest barbarism but peace, easy taxes, and a tolerable administration of justice: all the rest being brought about by the natural course of things." Sadly, many nations do not enjoy these three simple advantages.

CASE STUDY

The Colonial Origins of Modern Institutions

International data show a remarkable correlation between latitude and economic prosperity: nations closer to the equator typically have lower levels of income per person than nations farther from the equator. This fact is true in both the northern and southern hemispheres.

What explains the correlation? Some economists have suggested that the tropical climates near the equator have a direct negative impact on productivity. In the heat of the tropics, agriculture is more difficult, and disease is more prevalent. This makes the production of goods and services more difficult.

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[8]Rafael La Porta, Florencio Lopez-de-Silanes, Andrei Shleifer, and Robert Vishny, "Law and Finance," *Journal of Political Economy* 106 (1998): 1113–1155; Ross Levine and Robert G. King, "Finance and Growth: Schumpeter Might Be Right," *Quarterly Journal of Economics* 108 (1993): 717–737.

[9]Paulo Mauro, "Corruption and Growth," *Quarterly Journal of Economics* 110 (1995): 681–712.

Although the direct impact of geography is one reason tropical nations tend to be poor, it is not the whole story. Research by Daron Acemoglu, Simon Johnson, and James Robinson has suggested an indirect mechanism—the impact of geography on institutions. Here is their explanation, presented in several steps:

1. In the seventeenth, eighteenth, and nineteenth centuries, tropical climates presented European settlers with an increased risk of disease, especially malaria and yellow fever. As a result, when Europeans were colonizing much of the rest of the world, they avoided settling in tropical areas, such as most of Africa and Central America. The European settlers preferred areas with more moderate climates and better health conditions, such as the regions that are now the United States, Canada, and New Zealand.
2. In those areas where Europeans settled in large numbers, the settlers established European-like institutions that protected individual property rights and limited the power of government. By contrast, in tropical climates, the colonial powers often set up "extractive" institutions, including authoritarian governments, so they could take advantage of the area's natural resources. These institutions enriched the colonizers, but they did little to foster economic growth.
3. Although the era of colonial rule is now long over, the early institutions that the European colonizers established are strongly correlated with the modern institutions in the former colonies. In tropical nations, where the colonial powers set up extractive institutions, there is typically less protection of property rights even today. When the colonizers left, the extractive institutions remained and were simply taken over by new ruling elites.
4. The quality of institutions is a key determinant of economic performance. Where property rights are well protected, people have more incentive to make the investments that lead to economic growth. Where property rights are less respected, as is typically the case in tropical nations, investment and growth tend to lag behind.

This research suggests that much of the international variation in living standards that we observe today is a result of the long reach of history.[10] ■

Encouraging Technological Progress

The Solow model shows that sustained growth in income per worker must come from technological progress. The Solow model, however, takes technological progress as exogenous; it does not explain it. Unfortunately, the determinants of technological progress are not well understood.

Despite this limited understanding, many public policies are designed to stimulate technological progress. Most of these policies encourage the private sector to devote resources to technological innovation. For example, the patent system gives

[10]Daron Acemoglu, Simon Johnson, and James A. Robinson, "The Colonial Origins of Comparative Development: An Empirical Investigation," *American Economic Review* 91 (December 2001): 1369–1401.

a temporary monopoly to inventors of new products; the tax code offers tax breaks for firms engaging in research and development; and government agencies, such as the National Science Foundation, directly subsidize basic research in universities. In addition, as discussed above, proponents of industrial policy argue that the government should take a more active role in promoting specific industries that are key for rapid technological advance.

In recent years, the encouragement of technological progress has taken on an international dimension. Many of the companies that engage in research to advance technology are located in the United States and other developed nations. Developing nations such as China have an incentive to "free ride" on this research by not strictly enforcing intellectual property rights. That is, Chinese companies often use the ideas developed abroad without compensating the patent holders. The United States has strenuously objected to this practice, and China has promised to step up enforcement. If intellectual property rights were better enforced around the world, firms would have more incentive to engage in research, and this would promote worldwide technological progress.

CASE STUDY

The Worldwide Slowdown in Economic Growth

Beginning in the early 1970s, world policymakers faced a perplexing problem: a global slowdown in economic growth. Table 9-2 presents data on the growth in real GDP per person for the seven major economies. Growth in the United States fell from 2.2 percent before 1972 to 1.5 percent after 1972. Other countries experienced similar or more severe declines. Accumulated over many years, even a small change in the rate of growth has a large effect on economic

TABLE 9-2

Growth Around the World

	GROWTH IN OUTPUT PER PERSON (PERCENT PER YEAR)		
Country	**1948–1972**	**1972–1995**	**1995–2010**
Canada	2.9	1.8	1.6
France	4.3	1.6	1.1
West Germany	5.7	2.0	
Germany			1.3
Italy	4.9	2.3	0.6
Japan	8.2	2.6	0.6
United Kingdom	2.4	1.8	1.7
United States	2.2	1.5	1.5

Source: Angus Maddison, *Phases of Capitalist Development* (Oxford: Oxford University Press, 1982); *OECD National Accounts*; and *World Bank: World Development Indicators*.

well-being. Real income in the United States today is almost 25 percent lower than it would have been had growth remained at its previous level.

Why did this slowdown occur? Studies have shown that it was attributable to a fall in the rate at which the production function was improving over time. The appendix to this chapter explains how economists measure changes in the production function with a variable called *total factor productivity*, which is closely related to the efficiency of labor in the Solow model. There are many hypotheses to explain this fall in productivity growth. Here are four of them.

Measurement Problems One possibility is that the productivity slowdown did not really occur and that it shows up in the data because the data are flawed. As you may recall from Chapter 2, one problem in measuring inflation is correcting for changes in the quality of goods and services. The same issue arises when measuring output and productivity. For instance, if technological advance leads to *more* computers being built, then the increase in output and productivity is easy to measure. But if technological advance leads to *faster* computers being built, then output and productivity have increased, but that increase is more subtle and harder to measure. Government statisticians try to correct for changes in quality, but despite their best efforts, the resulting data are far from perfect.

Unmeasured quality improvements mean that our standard of living is rising more rapidly than the official data indicate. This issue should make us suspicious of the data, but by itself it cannot explain the productivity slowdown. To explain a *slowdown* in growth, one must argue that the measurement problems got *worse*. There is some indication that this might be so. As history passes, fewer people work in industries with tangible and easily measured output, such as agriculture, and more work in industries with intangible and less easily measured output, such as medical services. Yet few economists believe that measurement problems were the full story.

Oil Prices When the productivity slowdown began around 1973, the obvious hypothesis to explain it was the large increase in oil prices caused by the actions of the OPEC oil cartel. The primary piece of evidence was the timing: productivity growth slowed at the same time that oil prices skyrocketed. Over time, however, this explanation has appeared less likely. One reason is that the accumulated shortfall in productivity seems too large to be explained by an increase in oil prices; petroleum-based products are not that large a fraction of a typical firm's costs. In addition, if this explanation were right, productivity should have sped up when political turmoil in OPEC caused oil prices to plummet in 1986. Unfortunately, that did not happen.

Worker Quality Some economists suggest that the productivity slowdown might have been caused by changes in the labor force. In the early 1970s, the large baby-boom generation started leaving school and taking jobs. At the same time, changing social norms encouraged many women to leave full-time housework and enter the labor force. Both of these developments lowered the average level of experience among workers, which in turn lowered average productivity.

Other economists point to changes in worker quality as gauged by human capital. Although the educational attainment of the labor force continued to rise throughout this period, it was not increasing as rapidly as it had in the past.

Moreover, declining performance on some standardized tests suggests that the quality of education was declining. If so, this could explain slowing productivity growth.

The Depletion of Ideas Still other economists suggest that in the early 1970s the world started running out of new ideas about how to produce, pushing the economy into an age of slower technological progress. These economists often argue that the anomaly is not the period since 1970 but the preceding two decades. In the late 1940s, the economy had a large backlog of ideas that had not been fully implemented because of the Great Depression of the 1930s and World War II in the first half of the 1940s. After the economy used up this backlog, the argument goes, a slowdown in productivity growth was likely. Indeed, although the growth rates after 1972 were disappointing compared to those of the 1950s and 1960s, they were not lower than average growth rates from 1870 to 1950.

As any good doctor will tell you, sometimes a patient's illness goes away on its own, even if the doctor has failed to come up with a convincing diagnosis and remedy. This seems to be the outcome of the productivity slowdown. In the middle of the 1990s, economic growth took off, at least in the English-speaking countries of the United States, Canada, and the United Kingdom, in large part because of advances in computer and information technology, including the Internet. Yet this period of rapid growth was then offset by the financial crisis and deep recession in 2008–2009 (a topic we will discuss in Chapters 12 and 20). Overall, the period from 1995 to 2010 shows a continuation of the relatively slow growth experienced from 1972 to 1995.[11] ■

9-4 Beyond the Solow Model: Endogenous Growth Theory

> A chemist, a physicist, and an economist are all trapped on a desert island, trying to figure out how to open a can of food.
>
> "Let's heat the can over the fire until it explodes," says the chemist.
>
> "No, no," says the physicist, "let's drop the can onto the rocks from the top of a high tree."
>
> "I have an idea," says the economist. "First, we assume a can opener . . ."

This old joke takes aim at how economists use assumptions to simplify—and sometimes oversimplify—the problems they face. It is particularly apt when evaluating the theory of economic growth. One goal of growth theory is to explain the persistent rise in living standards that we observe in most parts of the world. The Solow growth model shows that such persistent growth must come from technological progress. But where does technological progress come from? In the Solow model, it is just assumed!

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[11] For various views on the growth slowdown, see "Symposium: The Slowdown in Productivity Growth" in the Fall 1988 issue of *The Journal of Economic Perspectives*. For a discussion of the subsequent growth acceleration and the role of information technology, see "Symposium: Computers and Productivity" in the Fall 2000 issue of *The Journal of Economic Perspectives*.

The preceding Case Study on the productivity slowdown of the 1970s and speed-up of the 1990s suggests that changes in the pace of technological progress are tremendously important. To fully understand the process of economic growth, we need to go beyond the Solow model and develop models that explain technological advance. Models that do this often go by the label **endogenous growth theory** because they reject the Solow model's assumption of exogenous technological change. Although the field of endogenous growth theory is large and sometimes complex, here we get a quick taste of this modern research.[12]

The Basic Model

To illustrate the idea behind endogenous growth theory, let's start with a particularly simple production function:

$$Y = AK,$$

where Y is output, K is the capital stock, and A is a constant measuring the amount of output produced for each unit of capital. Notice that this production function does not exhibit the property of diminishing returns to capital. One extra unit of capital produces A extra units of output, regardless of how much capital there is. This absence of diminishing returns to capital is the key difference between this endogenous growth model and the Solow model.

Now let's see what this production function says about economic growth. As before, we assume a fraction s of income is saved and invested. We therefore describe capital accumulation with an equation similar to those we used previously:

$$\Delta K = sY - \delta K.$$

This equation states that the change in the capital stock (ΔK) equals investment (sY) minus depreciation (δK). Combining this equation with the $Y = AK$ production function, we obtain, after a bit of manipulation,

$$\Delta Y/Y = \Delta K/K = sA - \delta.$$

This equation shows what determines the growth rate of output $\Delta Y/Y$. Notice that, as long as $sA > \delta$, the economy's income grows forever, even without the assumption of exogenous technological progress.

Thus, a simple change in the production function can dramatically alter the predictions about economic growth. In the Solow model, saving temporarily leads to growth, but diminishing returns to capital eventually force the economy to approach a steady state in which growth depends only on exogenous technological progress. By contrast, in this endogenous growth model, saving and investment can lead to persistent growth.

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[12]This section provides a brief introduction to the large and fascinating literature on endogenous growth theory. Early and important contributions to this literature include Paul M. Romer, "Increasing Returns and Long-Run Growth," *Journal of Political Economy* 94 (October 1986): 1002–1037; and Robert E. Lucas, Jr., "On the Mechanics of Economic Development," *Journal of Monetary Economics* 22 (1988): 3–42. The reader can learn more about this topic in the undergraduate textbook by David N. Weil, *Economic Growth,* 2nd ed. (Pearson, 2008).

But is it reasonable to abandon the assumption of diminishing returns to capital? The answer depends on how we interpret the variable K in the production function $Y = AK$. If we take the traditional view that K includes only the economy's stock of plants and equipment, then it is natural to assume diminishing returns. Giving 10 computers to a worker does not make that worker 10 times as productive as he or she is with one computer.

Advocates of endogenous growth theory, however, argue that the assumption of constant (rather than diminishing) returns to capital is more palatable if K is interpreted more broadly. Perhaps the best case can be made for the endogenous growth model by viewing knowledge as a type of capital. Clearly, knowledge is a key input into the economy's production—both its production of goods and services and its production of new knowledge. Compared to other forms of capital, however, it is less natural to assume that knowledge exhibits the property of diminishing returns. (Indeed, the increasing pace of scientific and technological innovation over the past few centuries has led some economists to argue that there are increasing returns to knowledge.) If we accept the view that knowledge is a type of capital, then this endogenous growth model with its assumption of constant returns to capital becomes a more plausible description of long-run economic growth.

A Two-Sector Model

Although the $Y = AK$ model is the simplest example of endogenous growth, the theory has gone well beyond this. One line of research has tried to develop models with more than one sector of production in order to offer a better description of the forces that govern technological progress. To see what we might learn from such models, let's sketch out an example.

The economy has two sectors, which we can call manufacturing firms and research universities. Firms produce goods and services, which are used for consumption and investment in physical capital. Universities produce a factor of production called "knowledge," which is then freely used in both sectors. The economy is described by the production function for firms, the production function for universities, and the capital-accumulation equation:

$$Y = F[K, (1 - u)LE] \qquad \text{(production function in manufacturing firms),}$$

$$\Delta E = g(u)E \qquad \text{(production function in research universities),}$$

$$\Delta K = sY - \delta K \qquad \text{(capital accumulation),}$$

where u is the fraction of the labor force in universities (and $1 - u$ is the fraction in manufacturing), E is the stock of knowledge (which in turn determines the efficiency of labor), and g is a function that shows how the growth in knowledge depends on the fraction of the labor force in universities. The rest of the notation is standard. As usual, the production function for the manufacturing firms is assumed to have constant returns to scale: if we double both the amount of physical capital (K) and the effective number of workers in manufacturing $[(1 - u)LE]$, we double the output of goods and services (Y).

This model is a cousin of the $Y = AK$ model. Most important, this economy exhibits constant (rather than diminishing) returns to capital, as long as capital is broadly defined to include knowledge. In particular, if we double both physical capital K and knowledge E, then we double the output of both sectors in the economy. As a result, like the $Y = AK$ model, this model can generate persistent growth without the assumption of exogenous shifts in the production function. Here persistent growth arises endogenously because the creation of knowledge in universities never slows down.

At the same time, however, this model is also a cousin of the Solow growth model. If u, the fraction of the labor force in universities, is held constant, then the efficiency of labor E grows at the constant rate $g(u)$. This result of constant growth in the efficiency of labor at rate g is precisely the assumption made in the Solow model with technological progress. Moreover, the rest of the model—the manufacturing production function and the capital-accumulation equation—also resembles the rest of the Solow model. As a result, for any given value of u, this endogenous growth model works just like the Solow model.

There are two key decision variables in this model. As in the Solow model, the fraction of output used for saving and investment, s, determines the steady-state stock of physical capital. In addition, the fraction of labor in universities, u, determines the growth in the stock of knowledge. Both s and u affect the level of income, although only u affects the steady-state growth rate of income. Thus, this model of endogenous growth takes a small step in the direction of showing which societal decisions determine the rate of technological change.

The Microeconomics of Research and Development

The two-sector endogenous growth model just presented takes us closer to understanding technological progress, but it still tells only a rudimentary story about the creation of knowledge. If one thinks about the process of research and development for even a moment, three facts become apparent. First, although knowledge is largely a public good (that is, a good freely available to everyone), much research is done in firms that are driven by the profit motive. Second, research is profitable because innovations give firms temporary monopolies, either because of the patent system or because there is an advantage to being the first firm on the market with a new product. Third, when one firm innovates, other firms build on that innovation to produce the next generation of innovations. These (essentially microeconomic) facts are not easily connected with the (essentially macroeconomic) growth models we have discussed so far.

Some endogenous growth models try to incorporate these facts about research and development. Doing this requires modeling both the decisions that firms face as they engage in research and the interactions among firms that have some degree of monopoly power over their innovations. Going into more detail about these models is beyond the scope of this book, but it should be clear already that one virtue of these endogenous growth models is that they offer a more complete description of the process of technological innovation.

One question these models are designed to address is whether, from the standpoint of society as a whole, private profit-maximizing firms tend to engage in too little or too much research. In other words, is the social return to research (which is what society cares about) greater or smaller than the private return (which is what motivates individual firms)? It turns out that, as a theoretical matter, there are effects in both directions. On the one hand, when a firm creates a new technology, it makes other firms better off by giving them a base of knowledge on which to build in future research. As Isaac Newton famously remarked, "If I have seen further, it is by standing on the shoulders of giants." On the other hand, when one firm invests in research, it can also make other firms worse off if it does little more than become the first to discover a technology that another firm would have invented in due course. This duplication of research effort has been called the "stepping on toes" effect. Whether firms left to their own devices do too little or too much research depends on whether the positive "standing on shoulders" externality or the negative "stepping on toes" externality is more prevalent.

Although theory alone is ambiguous about whether research effort is more or less than optimal, the empirical work in this area is usually less so. Many studies have suggested the "standing on shoulders" externality is important and, as a result, the social return to research is large—often in excess of 40 percent per year. This is an impressive rate of return, especially when compared to the return to physical capital, which we earlier estimated to be about 8 percent per year. In the judgment of some economists, this finding justifies substantial government subsidies to research.[13]

The Process of Creative Destruction

In his 1942 book *Capitalism, Socialism, and Democracy*, economist Joseph Schumpeter suggested that economic progress comes through a process of **creative destruction**. According to Schumpeter, the driving force behind progress is the entrepreneur with an idea for a new product, a new way to produce an old product, or some other innovation. When the entrepreneur's firm enters the market, it has some degree of monopoly power over its innovation; indeed, it is the prospect of monopoly profits that motivates the entrepreneur. The entry of the new firm is good for consumers, who now have an expanded range of choices, but it is often bad for incumbent producers, who may find it hard to compete with the entrant. If the new product is sufficiently better than old ones, the incumbents may even be driven out of business. Over time, the process keeps renewing itself. The entrepreneur's firm becomes an incumbent, enjoying high profitability until its product is displaced by another entrepreneur with the next generation of innovation.

History confirms Schumpeter's thesis that there are winners and losers from technological progress. For example, in England in the early nineteenth century, an important innovation was the invention and spread of machines that could produce textiles using unskilled workers at low cost. This technological advance was good for consumers, who could clothe themselves more cheaply. Yet skilled knitters in England saw their jobs threatened by new technology, and they responded by

[13]For an overview of the empirical literature on the effects of research, see Zvi Griliches, "The Search for R&D Spillovers," *Scandinavian Journal of Economics* 94 (1991): 29–47.

organizing violent revolts. The rioting workers, called Luddites, smashed the weaving machines used in the wool and cotton mills and set the homes of the mill owners on fire (a less than creative form of destruction). Today, the term "Luddite" refers to anyone who opposes technological progress.

A more recent example of creative destruction involves the retailing giant Walmart. Although retailing may seem like a relatively static activity, in fact it is a sector that has seen sizable rates of technological progress over the past several decades. Through better inventory-control, marketing, and personnel-management techniques, for example, Walmart has found ways to bring goods to consumers at lower cost than traditional retailers. These changes benefit consumers, who can buy goods at lower prices, and the stockholders of Walmart, who share in its profitability. But they adversely affect small mom-and-pop stores, which find it hard to compete when a Walmart opens nearby.

Faced with the prospect of being the victims of creative destruction, incumbent producers often look to the political process to stop the entry of new, more efficient competitors. The original Luddites wanted the British government to save their jobs by restricting the spread of the new textile technology; instead, Parliament sent troops to suppress the Luddite riots. Similarly, in recent years, local retailers have sometimes tried to use local land-use regulations to stop Walmart from entering their market. The cost of such entry restrictions, however, is a slower pace of technological progress. In Europe, where entry regulations are stricter than they are in the United States, the economies have not seen the emergence of retailing giants like Walmart; as a result, productivity growth in retailing has been much lower.[14]

Schumpeter's vision of how capitalist economies work has merit as a matter of economic history. Moreover, it has inspired some recent work in the theory of economic growth. One line of endogenous growth theory, pioneered by economists Philippe Aghion and Peter Howitt, builds on Schumpeter's insights by modeling technological advance as a process of entrepreneurial innovation and creative destruction.[15]

9-5 Conclusion

Long-run economic growth is the single most important determinant of the economic well-being of a nation's citizens. Everything else that macroeconomists study—unemployment, inflation, trade deficits, and so on—pales in comparison.

Fortunately, economists know quite a lot about the forces that govern economic growth. The Solow growth model and the more recent endogenous growth models show how saving, population growth, and technological progress interact in determining the level and growth of a nation's standard of living. These theories

[14]Robert J. Gordon, "Why Was Europe Left at the Station When America's Productivity Locomotive Departed?" *NBER Working Paper* No. 10661, 2004.

[15]Philippe Aghion and Peter Howitt, "A Model of Growth Through Creative Destruction," *Econometrica* 60 (1992): 323–351.

offer no magic recipe to ensure that an economy achieves rapid growth, but they give much insight, and they provide the intellectual framework for much of the debate over public policy aimed at promoting long-run economic growth.

Summary

1. In the steady state of the Solow growth model, the growth rate of income per person is determined solely by the exogenous rate of technological progress.
2. Many empirical studies have examined the extent to which the Solow model can help explain long-run economic growth. The model can explain much of what we see in the data, such as balanced growth and conditional convergence. Recent studies have also found that international variation in standards of living is attributable to a combination of capital accumulation and the efficiency with which capital is used.
3. In the Solow model with population growth and technological progress, the Golden Rule (consumption-maximizing) steady state is characterized by equality between the net marginal product of capital ($MPK - \delta$) and the steady-state growth rate of total income ($n + g$). In the U.S. economy, the net marginal product of capital is well in excess of the growth rate, indicating that the U.S. economy has a lower saving rate and less capital than it would have in the Golden Rule steady state.
4. Policymakers in the United States and other countries often claim that their nations should devote a larger percentage of their output to saving and investment. Increased public saving and tax incentives for private saving are two ways to encourage capital accumulation. Policymakers can also promote economic growth by setting up the appropriate legal and financial institutions to allocate resources efficiently and by ensuring proper incentives to encourage research and technological progress.
5. In the early 1970s, the rate of growth of income per person fell substantially in most industrialized countries, including the United States. The cause of this slowdown is not well understood. In the mid-1990s, the U.S. growth rate increased, most likely because of advances in information technology.
6. Modern theories of endogenous growth attempt to explain the rate of technological progress, which the Solow model takes as exogenous. These models try to explain the decisions that determine the creation of knowledge through research and development.

KEY CONCEPTS

Efficiency of labor

Labor-augmenting technological progress

Endogenous growth theory

Creative destruction

QUESTIONS FOR REVIEW

1. In the Solow model, what determines the steady-state rate of growth of income per worker?
2. In the steady state of the Solow model, at what rate does output per person grow? At what rate does capital per person grow? How does this compare with the U.S. experience?
3. What data would you need to determine whether an economy has more or less capital than in the Golden Rule steady state?
4. How can policymakers influence a nation's saving rate?
5. What has happened to the rate of productivity growth over the past 50 years? How might you explain this phenomenon?
6. How does endogenous growth theory explain persistent growth without the assumption of exogenous technological progress? How does this differ from the Solow model?

PROBLEMS AND APPLICATIONS

1. Suppose an economy described by the Solow model has the following production function:

$$Y = K^{1/2}(LE)^{1/2}.$$

   a. For this economy, what is $f(k)$?

   b. Use your answer to part (a) to solve for the steady-state value of y as a function of s, n, g, and δ.

   c. Two neighboring economies have the above production function, but they have different parameter values. Atlantis has a saving rate of 28 percent and a population growth rate of 1 percent per year. Xanadu has a saving rate of 10 percent and a population growth rate of 4 percent per year. In both countries, $g = 0.02$ and $\delta = 0.04$. Find the steady-state value of y for each country.

2. In the United States, the capital share of GDP is about 30 percent, the average growth in output is about 3 percent per year, the depreciation rate is about 4 percent per year, and the capital–output ratio is about 2.5. Suppose that the production function is Cobb–Douglas, so that the capital share in output is constant, and that the United States has been in a steady state. (For a discussion of the Cobb–Douglas production function, see Chapter 3.)

   a. What must the saving rate be in the initial steady state? [*Hint:* Use the steady-state relationship, $sy = (\delta + n + g)k$.]

   b. What is the marginal product of capital in the initial steady state?

   c. Suppose that public policy raises the saving rate so that the economy reaches the Golden Rule level of capital. What will the marginal product of capital be at the Golden Rule steady state? Compare the marginal product at the Golden Rule steady state to the marginal product in the initial steady state. Explain.

   d. What will the capital–output ratio be at the Golden Rule steady state? (*Hint:* For the Cobb–Douglas production function, the capital–output ratio is related to the marginal product of capital.)

   e. What must the saving rate be to reach the Golden Rule steady state?

3. Prove each of the following statements about the steady state of the Solow model with population growth and technological progress.

   a. The capital–output ratio is constant.

   b. Capital and labor each earn a constant share of an economy's income. [*Hint:* Recall the definition $MPK = f(k + 1) - f(k)$.]

   c. Total capital income and total labor income both grow at the rate of population growth plus the rate of technological progress, $n + g$.

   d. The real rental price of capital is constant, and the real wage grows at the rate of technological progress g. (*Hint:* The real rental price of capital equals total capital income divided by the capital stock, and the real wage equals total labor income divided by the labor force.)

4. Two countries, Richland and Poorland, are described by the Solow growth model. They have the same Cobb–Douglas production function, $F(K, L) = A\ K^{\alpha}L^{1-\alpha}$, but with different quantities of capital and labor. Richland saves 32 percent of its income, while Poorland saves 10 percent. Richland has population growth of 1 percent per year, while Poorland has population growth of 3 percent. (The numbers in this problem are chosen to be approximately realistic descriptions of rich and poor nations.) Both nations have technological progress at a rate of 2 percent per year and depreciation at a rate of 5 percent per year.

a. What is the per-worker production function $f(k)$?

b. Solve for the ratio of Richland's steady-state income per worker to Poorland's. (*Hint:* The parameter α will play a role in your answer.)

c. If the Cobb–Douglas parameter α takes the conventional value of about 1/3, how much higher should income per worker be in Richland compared to Poorland?

d. Income per worker in Richland is actually 16 times income per worker in Poorland. Can you explain this fact by changing the value of the parameter α? What must it be? Can you think of any way of justifying such a value for this parameter? How else might you explain the large difference in income between Richland and Poorland?

5. The amount of education the typical person receives varies substantially among countries. Suppose you were to compare a country with a highly educated labor force and a country with a less educated labor force. Assume that education affects only the level of the efficiency of labor. Also assume that the countries are otherwise the same: they have the same saving rate, the same depreciation rate, the same population growth rate, and the same rate of technological progress. Both countries are described by the Solow model and are in their steady states. What would you predict for the following variables?

a. The rate of growth of total income

b. The level of income per worker

c. The real rental price of capital

d. The real wage

6. This question asks you to analyze in more detail the two-sector endogenous growth model presented in the text.

a. Rewrite the production function for manufactured goods in terms of output per effective worker and capital per effective worker.

b. In this economy, what is break-even investment (the amount of investment needed to keep capital per effective worker constant)?

c. Write down the equation of motion for k, which shows Δk as saving minus break-even investment. Use this equation to draw a graph showing the determination of steady-state k. (*Hint:* This graph will look much like those we used to analyze the Solow model.)

d. In this economy, what is the steady-state growth rate of output per worker Y/L? How do the saving rate s and the fraction of the labor force in universities u affect this steady-state growth rate?

e. Using your graph, show the impact of an increase in u. (*Hint:* This change affects both curves.) Describe both the immediate and the steady-state effects.

f. Based on your analysis, is an increase in u an unambiguously good thing for the economy? Explain.

7. Choose two countries that interest you—one rich and one poor. What is the income per person in each country? Find some data on country characteristics that might help explain the difference in income: investment rates, population growth rates, educational attainment, and so on. (*Hint*: The Web site of the World Bank, www.worldbank.org, is one place to find such data.) How might you figure out which of these factors is most responsible for the observed income difference? In your judgment, how useful is the Solow model as an analytic tool for understanding the difference between the two countries you chose?

APPENDIX

Accounting for the Sources of Economic Growth

Real GDP in the United States has grown an average of about 3 percent per year over the past 50 years. What explains this growth? In Chapter 3 we linked the output of the economy to the factors of production—capital and labor—and to the production technology. Here we develop a technique called *growth accounting* that divides the growth in output into three different sources: increases in capital, increases in labor, and advances in technology. This breakdown provides us with a measure of the rate of technological change.

Increases in the Factors of Production

We first examine how increases in the factors of production contribute to increases in output. To do this, we start by assuming there is no technological change, so the production function relating output Y to capital K and labor L is constant over time:

$$Y = F(K, L).$$

In this case, the amount of output changes only because the amount of capital or labor changes.

Increases in Capital First, consider changes in capital. If the amount of capital increases by ΔK units, by how much does the amount of output increase? To answer this question, we need to recall the definition of the marginal product of capital MPK:

$$MPK = F(K + 1, L) - F(K, L).$$

The marginal product of capital tells us how much output increases when capital increases by 1 unit. Therefore, when capital increases by ΔK units, output increases by approximately $MPK \times \Delta K$.[16]

For example, suppose that the marginal product of capital is 1/5; that is, an additional unit of capital increases the amount of output produced by one-fifth of a unit. If we increase the amount of capital by 10 units, we can compute the amount of additional output as follows:

$$\begin{aligned} \Delta Y &= \qquad MPK \qquad \times \qquad \Delta K \\ &= 1/5 \ \frac{\text{units of output}}{\text{unit of capital}} \times 10 \text{ units of capital} \\ &= 2 \text{ units of output.} \end{aligned}$$

[16]Note the word "approximately" here. This answer is only an approximation because the marginal product of capital varies: it falls as the amount of capital increases. An exact answer would take into account the fact that each unit of capital has a different marginal product. If the change in K is not too large, however, the approximation of a constant marginal product is very accurate.

By increasing capital by 10 units, we obtain 2 more units of output. Thus, we use the marginal product of capital to convert changes in capital into changes in output.

Increases in Labor Next, consider changes in labor. If the amount of labor increases by ΔL units, by how much does output increase? We answer this question the same way we answered the question about capital. The marginal product of labor MPL tells us how much output changes when labor increases by 1 unit—that is,

$$MPL = F(K, L + 1) - F(K, L).$$

Therefore, when the amount of labor increases by ΔL units, output increases by approximately $MPL \times \Delta L$.

For example, suppose that the marginal product of labor is 2; that is, an additional unit of labor increases the amount of output produced by 2 units. If we increase the amount of labor by 10 units, we can compute the amount of additional output as follows:

$$\begin{aligned} \Delta Y &= \quad MPL \quad \times \quad \Delta L \\ &= 2\,\frac{\text{units of output}}{\text{unit of labor}} \times 10 \text{ units of labor} \\ &= 20 \text{ units of output.} \end{aligned}$$

By increasing labor by 10 units, we obtain 20 more units of output. Thus, we use the marginal product of labor to convert changes in labor into changes in output.

Increases in Capital and Labor Finally, let's consider the more realistic case in which both factors of production change. Suppose that the amount of capital increases by ΔK and the amount of labor increases by ΔL. The increase in output then comes from two sources: more capital and more labor. We can divide this increase into the two sources using the marginal products of the two inputs:

$$\Delta Y = (MPK \times \Delta K) + (MPL \times \Delta L).$$

The first term in parentheses is the increase in output resulting from the increase in capital; the second term in parentheses is the increase in output resulting from the increase in labor. This equation shows us how to attribute growth to each factor of production.

We now want to convert this last equation into a form that is easier to interpret and apply to the available data. First, with some algebraic rearrangement, the equation becomes[17]

$$\frac{\Delta Y}{Y} = \left(\frac{MPK \times K}{Y}\right)\frac{\Delta K}{K} + \left(\frac{MPL \times L}{Y}\right)\frac{\Delta L}{L}.$$

[17] *Mathematical note:* To see that this is equivalent to the previous equation, note that we can multiply both sides of this equation by Y and thereby cancel Y from three places in which it appears. We can cancel the K in the top and bottom of the first term on the right-hand side and the L in the top and bottom of the second term on the right-hand side. These algebraic manipulations turn this equation into the previous one.

This form of the equation relates the growth rate of output, $\Delta Y/Y$, to the growth rate of capital, $\Delta K/K$, and the growth rate of labor, $\Delta L/L$.

Next, we need to find some way to measure the terms in parentheses in the last equation. In Chapter 3 we showed that the marginal product of capital equals its real rental price. Therefore, $MPK \times K$ is the total return to capital, and $(MPK \times K)/Y$ is capital's share of output. Similarly, the marginal product of labor equals the real wage. Therefore, $MPL \times L$ is the total compensation that labor receives, and $(MPL \times L)/Y$ is labor's share of output. Under the assumption that the production function has constant returns to scale, Euler's theorem (which we discussed in Chapter 3) tells us that these two shares sum to 1. In this case, we can write

$$\frac{\Delta Y}{Y} = \alpha\frac{\Delta K}{K} + (1 - \alpha)\frac{\Delta L}{L},$$

where α is capital's share and $(1 - \alpha)$ is labor's share.

This last equation gives us a simple formula for showing how changes in inputs lead to changes in output. It shows, in particular, that we must weight the growth rates of the inputs by the factor shares. As we discussed in Chapter 3, capital's share in the United States is about 30 percent, that is, $\alpha = 0.30$. Therefore, a 10 percent increase in the amount of capital ($\Delta K/K = 0.10$) leads to a 3 percent increase in the amount of output ($\Delta Y/Y = 0.03$). Similarly, a 10 percent increase in the amount of labor ($\Delta L/L = 0.10$) leads to a 7 percent increase in the amount of output ($\Delta Y/Y = 0.07$).

Technological Progress

So far in our analysis of the sources of growth, we have been assuming that the production function does not change over time. In practice, of course, technological progress improves the production function. For any given amount of inputs, we can produce more output today than we could in the past. We now extend the analysis to allow for technological progress.

We include the effects of the changing technology by writing the production function as

$$Y = AF(K, L),$$

where A is a measure of the current level of technology called *total factor productivity*. Output now increases not only because of increases in capital and labor but also because of increases in total factor productivity. If total factor productivity increases by 1 percent and if the inputs are unchanged, then output increases by 1 percent.

Allowing for a changing level of technology adds another term to our equation accounting for economic growth:

$$\begin{array}{ccccccc} \dfrac{\Delta Y}{Y} & = & \alpha\dfrac{\Delta K}{K} & + & (1 - \alpha)\dfrac{\Delta L}{L} & + & \dfrac{\Delta A}{A} \\ \text{Growth in Output} & = & \text{Contribution of Capital} & + & \text{Contribution of Labor} & + & \text{Growth in Total Factor Productivity}. \end{array}$$

This is the key equation of growth accounting. It identifies and allows us to measure the three sources of growth: changes in the amount of capital, changes in the amount of labor, and changes in total factor productivity.

Because total factor productivity is not directly observable, it is measured indirectly. We have data on the growth in output, capital, and labor; we also have data on capital's share of output. From these data and the growth-accounting equation, we can compute the growth in total factor productivity to make sure that everything adds up:

$$\frac{\Delta A}{A} = \frac{\Delta Y}{Y} - \alpha\frac{\Delta K}{K} - (1 - \alpha)\frac{\Delta L}{L}.$$

$\Delta A/A$ is the change in output that cannot be explained by changes in inputs. Thus, the growth in total factor productivity is computed as a residual—that is, as the amount of output growth that remains after we have accounted for the determinants of growth that we can measure directly. Indeed, $\Delta A/A$ is sometimes called the *Solow residual,* after Robert Solow, who first showed how to compute it.[18]

Total factor productivity can change for many reasons. Changes most often arise because of increased knowledge about production methods, so the Solow residual is often used as a measure of technological progress. Yet other factors, such as education and government regulation, can affect total factor productivity as well. For example, if higher public spending raises the quality of education, then workers may become more productive and output may rise, which implies higher total factor productivity. As another example, if government regulations require firms to purchase capital to reduce pollution or increase worker safety, then the capital stock may rise without any increase in measured output, which implies lower total factor productivity. *Total factor productivity captures anything that changes the relation between measured inputs and measured output.*

The Sources of Growth in the United States

Having learned how to measure the sources of economic growth, we now look at the data. Table 9-3 uses U.S. data to measure the contributions of the three sources of growth between 1948 and 2010.

This table shows that output in the non-farm business sector grew an average of 3.4 percent per year during this time. Of this 3.4 percent, 1.0 percent was attributable to increases in the capital stock, 1.2 percent to increases in the labor input, and 1.2 percent to increases in total factor productivity. These data show that increases in capital, labor, and productivity have contributed almost equally to economic growth in the United States.

[18]Robert M. Solow, "Technical Change and the Aggregate Production Function," *Review of Economics and Statistics* 39 (1957): 312–320. It is natural to ask how growth in labor efficiency E relates to growth in total factor productivity. One can show that $\Delta A/A = (1 - \alpha)\Delta E/E$, where α is capital's share. Thus, technological change as measured by growth in the efficiency of labor is proportional to technological change as measured by the Solow residual.

TABLE 9-3

Accounting for Economic Growth in the United States

			SOURCES OF GROWTH				
Years	**Output Growth** $\Delta Y/Y$	=	**Capital** $\alpha\Delta K/K$	+	**Labor** $(1-\alpha)\Delta L/L$	+	**Total Factor Productivity** $\Delta A/A$
			(average percentage increase per year)				
1948–2010	3.4		1.0		1.2		1.2
1948–1972	4.1		1.0		1.2		1.9
1972–1995	3.4		1.4		1.3		0.7
1995–2010	2.8		0.4		1.1		1.3

Source: US Department of Labor. Data are for the non-farm business sector.

Table 9-3 also shows that the growth in total factor productivity slowed substantially during the period from 1972 to 1995. In a Case Study in this chapter, we discussed some hypotheses to explain this productivity slowdown.

CASE STUDY

Growth in the East Asian Tigers

Perhaps the most spectacular growth experiences in recent history have been those of the "Tigers" of East Asia: Hong Kong, Singapore, South Korea, and Taiwan. From 1966 to 1990, while real income per person was growing about 2 percent per year in the United States, it grew more than 7 percent per year in each of these countries. In the course of a single generation, real income per person increased fivefold, moving the Tigers from among the world's poorest countries to among the richest. (In the late 1990s, a period of pronounced financial turmoil tarnished the reputation of some of these economies. But this short-run problem, which we examine in a Case Study in Chapter 13 doesn't come close to reversing the spectacular long-run growth that the Asian Tigers have experienced.)

What accounts for these growth miracles? Some commentators have argued that the success of these four countries is hard to reconcile with basic growth theory, such as the Solow growth model, which takes technology as growing at a constant, exogenous rate. They have suggested that these countries' rapid growth is explained by their ability to imitate foreign technologies. By adopting technology developed abroad, the argument goes, these countries managed to improve their production functions substantially in a relatively short period of time. If this argument is correct, these countries should have experienced unusually rapid growth in total factor productivity.

One study shed light on this issue by examining in detail the data from these four countries. The study found that their exceptional growth can be traced to large increases in measured factor inputs: increases in labor-force participation, increases in the capital stock, and increases in educational attainment. In South Korea, for example, the investment–GDP ratio rose from about 5 percent in the 1950s to about 30 percent in the 1980s; the percentage of the working population with at least a high school education went from 26 percent in 1966 to 75 percent in 1991.

Once we account for growth in labor, capital, and human capital, little of the growth in output is left to explain. None of these four countries experienced unusually rapid growth in total factor productivity. Indeed, the average growth in total factor productivity in the East Asian Tigers was almost exactly the same as in the United States. Thus, although these countries' rapid growth has been truly impressive, it is easy to explain using the tools of basic growth theory.[19] ■

The Solow Residual in the Short Run

When Robert Solow introduced his famous residual, his aim was to shed light on the forces that determine technological progress and economic growth in the long run. But economist Edward Prescott has looked at the Solow residual as a measure of technological change over shorter periods of time. He concludes that fluctuations in technology are a major source of short-run changes in economic activity.

Figure 9-2 shows the Solow residual and the growth in output using annual data for the United States during the period 1960 to 2010. Notice that the Solow residual fluctuates substantially. If Prescott's interpretation is correct, then we can draw conclusions from these short-run fluctuations, such as that technology worsened in 1982 and improved in 1984. Notice also that the Solow residual moves closely with output: in years when output falls, technology tends to worsen. In Prescott's view, this fact implies that recessions are driven by adverse shocks to technology. The hypothesis that technological shocks are the driving force behind short-run economic fluctuations, and the complementary hypothesis that monetary policy has no role in explaining these fluctuations, is the foundation for an approach called *real-business-cycle theory*.

Prescott's interpretation of these data is controversial, however. Many economists believe that the Solow residual does not accurately represent changes in technology over short periods of time. The standard explanation of the cyclical behavior of the Solow residual is that it results from two measurement problems.

First, during recessions, firms may continue to employ workers they do not need so that they will have these workers on hand when the economy recovers. This phenomenon, called *labor hoarding*, means that labor input is overestimated in recessions because the hoarded workers are probably not working as hard as usual. As a result, the Solow residual is more cyclical than the available production technology. In a recession, productivity as measured by the Solow residual falls even if technology has not changed simply because hoarded workers are sitting around waiting for the recession to end.

Second, when demand is low, firms may produce things that are not easily measured. In recessions, workers may clean the factory, organize the inventory, get

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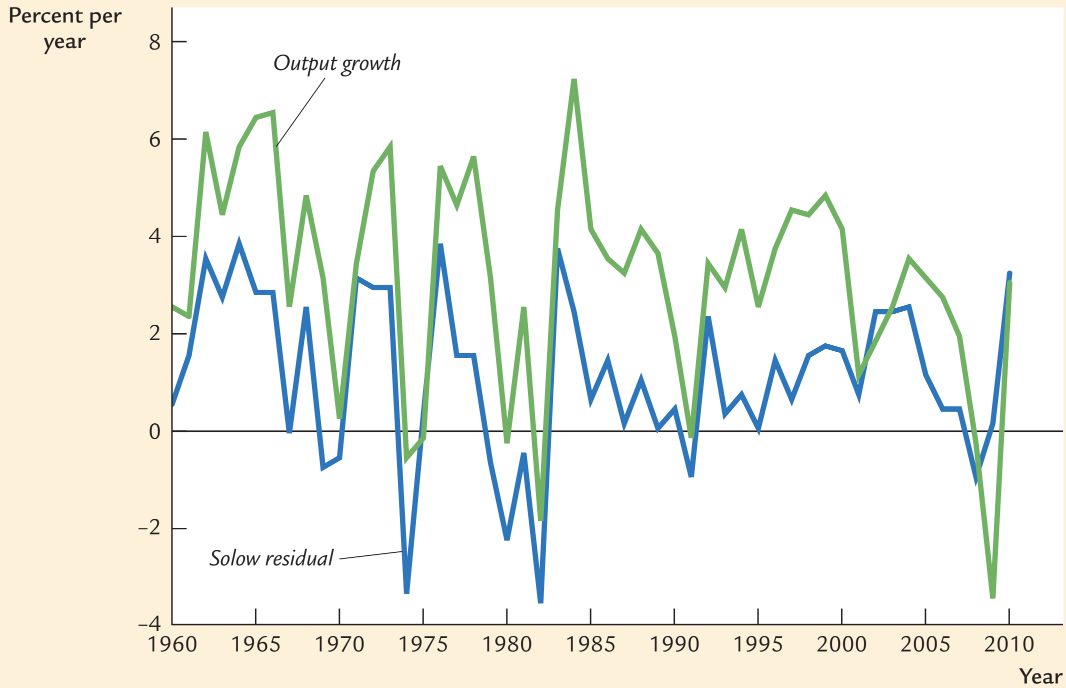
[19] Alwyn Young, "The Tyranny of Numbers: Confronting the Statistical Realities of the East Asian Growth Experience," *Quarterly Journal of Economics* 101 (August 1995): 641–680.

FIGURE 9-2



Growth in Output and the Solow Residual The Solow residual, which some economists interpret as a measure of technology shocks, fluctuates with the economy's output of goods and services.

Sources: U.S. Department of Commerce, U.S. Department of Labor, and author's calculations.

some training, and do other useful tasks that standard measures of output fail to include. If so, then output is underestimated in recessions, which would also make the measured Solow residual cyclical for reasons other than technology.

Thus, economists can interpret the cyclical behavior of the Solow residual in different ways. Some economists point to the low productivity in recessions as evidence for adverse technology shocks. Others believe that measured productivity is low in recessions because workers are not working as hard as usual and because more of their output is not measured. Unfortunately, there is no clear evidence on the importance of labor hoarding and the cyclical mismeasurement of output. Therefore, different interpretations of Figure 9-2 persist.[20]

[20]To read more about this topic, see Edward C. Prescott, "Theory Ahead of Business Cycle Measurement," and Lawrence H. Summers, "Some Skeptical Observations on Real Business Cycle Theory," both in *Quarterly Review*, Federal Reserve Bank of Minneapolis (Fall 1986); N. Gregory Mankiw, "Real Business Cycles: A New Keynesian Perspective," *Journal of Economic Perspectives* 3 (Summer 1989): 79–90; Bennett T. McCallum, "Real Business Cycle Models," in R. Barro, ed., *Modern Business Cycle Theory* (Cambridge, Mass.: Harvard University Press, 1989), 16–50; and Charles I. Plosser, "Understanding Real Business Cycles," *Journal of Economic Perspectives* 3 (Summer 1989): 51–77.

MORE PROBLEMS AND APPLICATIONS

1. In the economy of Solovia, the owners of capital get two-thirds of national income, and the workers receive one-third.

a. The men of Solovia stay at home performing household chores, while the women work in factories. If some of the men started working outside the home so that the labor force increased by 5 percent, what would happen to the measured output of the economy? Does labor productivity—defined as output per worker—increase, decrease, or stay the same? Does total factor productivity increase, decrease, or stay the same?

b. In year 1, the capital stock was 6, the labor input was 3, and output was 12. In year 2, the capital stock was 7, the labor input was 4, and output was 14. What happened to total factor productivity between the two years?

2. Labor productivity is defined as Y/L, the amount of output divided by the amount of labor input. Start with the growth-accounting equation and show that the growth in labor productivity depends on growth in total factor productivity and growth in the capital–labor ratio. In particular, show that

$$\frac{\Delta(Y/L)}{Y/L} = \frac{\Delta A}{A} + \alpha\frac{\Delta(K/L)}{K/L}.$$

Hint: You may find the following mathematical trick helpful. If $z = wx$, then the growth rate of z is approximately the growth rate of w plus the growth rate of x. That is,

$$\Delta z/z \approx \Delta w/w + \Delta x/x.$$

3. Suppose an economy described by the Solow model is in a steady state with population growth n of 1.8 percent per year and technological progress g of 1.8 percent per year. Total output and total capital grow at 3.6 percent per year. Suppose further that the capital share of output is 1/3. If you used the growth-accounting equation to divide output growth into three sources—capital, labor, and total factor productivity—how much would you attribute to each source? Compare your results to the figures we found for the United States in Table 9-3.