Time Series

A Simple Index Number

$$I_t = \frac{Y_t}{Y_0} \cdot 100$$

A Laspeyres Index

$$I_t = \frac{\sum\limits_{i=1}^{k} Q_{it_0} P_{it}}{\sum\limits_{i=1}^{k} Q_{it_0} P_{it_0}} \cdot 100$$

A Paasche Index

$$I_t = \frac{\sum\limits_{i=1}^{k} Q_{it} P_{it}}{\sum\limits_{i=1}^{k} Q_{it} P_{it_0}} \cdot 100$$

Exponential Smoothing ($0 \le w \le 1$)

$$E_1 = Y_1$$

$$E_2 = wY_2 + (1-w)E_1$$

$$E_3 = wY_3 + (1-w)E_2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..$$

$$E_t = wY_t + (1-w)E_{t-1}$$

Time Series Components

$$Y_t = T_t + C_t + S_t + R_t,$$

where T_t – secular trend, C_t – cyclical effect, S_t – seasonal effect and R_t – residual effect.

Forecasting: Exponential Smoothing

$$F_{t+1} = E_t$$

$$F_{t+2} = F_{t+1}$$

$$F_{t+3} = F_{t+3} = F_{t+1}, \quad \text{etc.}$$

Holt – Winters (Double Exponential) Smoothing ($0 \le \nu \le 1$)

$$E_2 = Y_2; \quad T_2 = Y_2 - Y_1$$

$$E_3 = wY_3 + (1-w)(E_2 + T_2); \quad T_3 = \nu(E_3 - E_2) + (1-\nu)T_2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..$$

$$E_t = wY_t + (1-w)(E_{t-1} + T_{t-1}); \quad T_t = \nu(E_t - E_{t-1}) + (1-\nu)T_{t-1}$$

Forecasting: Holt – Winters Smoothing model

$$F_{t+1} = E_t + T_t$$

$$F_{t+k} = E_t + kT_t$$

Measuring Forecast Accuracy

1. Mean Absolute Deviation

$$MAD = \frac{\sum_{t=1}^{m} |Y_t - F_t|}{m}$$

2. Mean Absolute Percentage Error

$$MAPE = \frac{\sum_{t=1}^{m} \left|\frac{Y_t - F_t}{Y_t}\right|}{m} \cdot 100$$

3. Root Mean Squared Error

$$RMSE = \sqrt{\frac{\sum_{t=1}^{m} (Y_t - F_t)^2}{m}}$$

Seasonal Regression Model

$$E(Y_t) = \beta_0 + \beta_1 t + \beta_2 Q_1 + \beta_3 Q_2 + \beta_4 Q_3,$$

where t – time period and

$$Q_1 = \begin{cases} 1, & \text{if quarter } 1 \\ 0, & \text{if quarter } 2, 3, \text{ or } 4 \end{cases}$$

$$Q_2 = \begin{cases} 1, & \text{if quarter } 2 \\ 0, & \text{if quarter } 1, 3, \text{ or } 4 \end{cases}$$

$$Q_3 = \begin{cases} 1, & \text{if quarter } 3 \\ 0, & \text{if quarter } 1, 2, \text{ or } 4 \end{cases}$$