Properties of the Multinomial Experiment

1. The experiment consists of n identical trials.

2. There are k possible outcomes to each trial. These outcomes are called *classes, categories*, or *cells.*

3. The probabilities of the k outcomes, denoted by p_1, p_2, ..., p_k, remain the same from trial to trial, where $p_1 + p_2 + ... + p_k = 1$.

4. The trials are independent.

5. The random variables of interest are the *cell counts*, n_1, n_2, ..., n_k, of the number of observations that fall in each of the k classes.

A Test of a Hypothesis about Multinomial Probabilities: One–Way Table

$H_0: \ p_1 = p_{1,0}, \ p_2 = p_{2,0}, ..., \ p_k = p_{k,0}$

where $p_{1,0}, \ p_{2,0}, ..., \ p_{k,0}$ represent the hypothesized values of the multinomial probabilities

H_a: At least one of the multinomial probabilities does not equal its hypothesized value

Test statistic: $\chi^2 = \sum \frac{[n_i - E(n_i)]^2}{E(n_i)}$

where $E(n_i) = np_{i,0}$ is the *expected cell count*, that is, the expected number of outcomes of type i assuming that H_0 is true. The total sample size is n.

Rejection region: $\chi^2 > \chi^2_\alpha$,

where χ^2_α has $(k-1)$ df.

Conditions Required for a Valid χ^2 Test: One–Way Table

1. A multinomial experiment has been conducted. This is generally satisfied by taking a random sample from the population of interest.

2. The sample size n is large. This is satisfied if for every cell, the expected cell count $E(n_i)$ will be equal to 5 or more.

Finding Expected Cell Counts for a Two-Way Contingency Table

The estimate of the expected number of observations falling into the cell in row i and column j is given by

$$E_{ij} = \frac{R_i C_j}{n}$$

where $R_i =$ total for row i, $C_j =$ total for column j, and $n =$ sample size.

General Form of a Contingency Table Analysis: A χ^2-Test for Independence

$H_0:$ The two classifications are independent

$H_a:$ The two classifications are dependent

Test statistic: $\chi^2 = \sum \frac{[n_{ij} - E_{ij}]^2}{E_{ij}}$,

where $E_{ij} = \frac{R_i C_j}{n}$.

Rejection region: $\chi^2 > \chi^2_\alpha$,

where χ^2_α has $(r-1)(c-1)$ df.

Conditions Required for a Valid χ^2-Test: Contingency Table

1. The n observed counts are a random sample from the population of interest. We may then consider this to be a multinomial experiment with $r \times c$ possible outcomes.

2. The sample size, n, will be large enough so that, for every cell, the expected count, E_{ij}, will be equal to 5 or more.