**SOLUTIONS QUESTION 1**

## Problems and Applications

1. a. To solve for the steady-state value of $y$ as a function of $s$, $n$, $g$, and $\delta$, we begin with the equation for the change in the capital stock in the steady state:

$$\Delta k = sf(k) - (\delta + n + g)k = 0.$$

The production function $y = \sqrt{k}$ can also be rewritten as $y^2 = k$. Plugging this production function into the equation for the change in the capital stock, we find that in the steady state:

$$sy - (\delta + n + g)y^2 = 0.$$

Solving this, we find the steady-state value of $y$:

$$y^* = s/(\delta + n + g).$$

b. The question provides us with the following information about each country:

| | | | |
|---|---|---|---|
| Developed country: | $s = 0.28$ | Less-developed country: | $s = 0.10$ |
| | $n = 0.01$ | | $n = 0.04$ |
| | $g = 0.02$ | | $g = 0.02$ |
| | $\delta = 0.04$ | | $\delta = 0.04$ |

Using the equation for $y^*$ that we derived in part (a), we can calculate the steady-state values of $y$ for each country.

Developed country: $y^* = 0.28/(0.04 + 0.01 + 0.02) = 4.$

Less-developed country: $y^* = 0.10/(0.04 + 0.04 + 0.02) = 1.$

c. The equation for $y^*$ that we derived in part (a) shows that the less-developed country could raise its level of income by reducing its population growth rate $n$ or by increasing its saving rate $s$. Policies that reduce population growth include introducing methods of birth control and implementing disincentives for having children. Policies that increase the saving rate include increasing public saving by reducing the budget deficit and introducing private saving incentives such as I.R.A.'s and other tax concessions that increase the return to saving.

## More Problems and Applications to Chapter 8

1. a. The growth in total output ($Y$) depends on the growth rates of labor ($L$), capital ($K$), and total factor productivity ($A$), as summarized by the equation:

$$\Delta Y/Y = \alpha \Delta K/K + (1 - \alpha)\Delta L/L + \Delta A/A,$$

where $\alpha$ is capital's share of output. We can look at the effect on output of a 5-percent increase in labor by setting $\Delta K/K = \Delta A/A = 0$. Since $\alpha = 2/3$, this gives us

$$\begin{aligned}\Delta Y/Y &= (1/3)\,(5\%)\\ &= 1.67\%.\end{aligned}$$

A 5-percent increase in labor input increases output by 1.67 percent.

Labor productivity is $Y/L$. We can write the growth rate in labor productivity as

$$\frac{\Delta(Y/L)}{Y/L} = \frac{\Delta Y}{Y} - \frac{\Delta L}{L}.$$

Substituting for the growth in output and the growth in labor, we find

$$\begin{aligned}\Delta(Y/L)/(Y/L) &= 1.67\% - 5.0\%\\ &= -3.34\%.\end{aligned}$$

Labor productivity falls by 3.34 percent.

To find the change in total factor productivity, we use the equation

$$\Delta A/A = \Delta Y/Y - \alpha\Delta K/K - (1 - \alpha)\Delta L/L.$$

For this problem, we find

$$\begin{aligned}\Delta A/A &= 1.67\% - 0 - (1/3)\,(5\%)\\ &= 0.\end{aligned}$$

Total factor productivity is the amount of output growth that remains after we have accounted for the determinants of growth that we can measure. In this case, there is no change in technology, so all of the output growth is attributable to measured input growth. That is, total factor productivity growth is zero, as expected.

b. Between years 1 and 2, the capital stock grows by 1/6, labor input grows by 1/3, and output grows by 1/6. We know that the growth in total factor productivity is given by

$$\Delta A/A = \Delta Y/Y - \alpha\Delta K/K - (1 - \alpha)\Delta L/L.$$

Substituting the numbers above, and setting $\alpha = 2/3$, we find

$$\begin{aligned}\Delta A/A &= (1/6) - (2/3)(1/6) - (1/3)(1/3)\\ &= 3/18 - 2/18 - 2/18\\ &= -\,1/18\\ &= -\,.056.\end{aligned}$$

Total factor productivity falls by 1/18, or approximately 5.6 percent.

By definition, output Y equals labor productivity $Y/L$ multiplied by the labor force $L$:

$$Y = (Y/L)L.$$

Using the mathematical trick in the hint, we can rewrite this as

$$\frac{\Delta Y}{Y} = \frac{\Delta(Y/L)}{Y/L} + \frac{\Delta L}{L}.$$

We can rearrange this as

$$\frac{\Delta(Y/L)}{Y/L} = \frac{\Delta Y}{Y} - \frac{\Delta L}{L}.$$