Lecture 9

**Derivatives of Implicit Functions**

if 2 expressions are identified equal, their respective total differentials must be equal

(1)

we can substitute this dy into (1)

(3)

(for all i)

(i=i,m)

for simple case F(y,x)=0,, at the point around when the implicit functiom

Example 1

Example 2

Example 3

;

Example 4

-?

Extension to the Simultaneous – Equation case

(8.24) Set of simultaneous equations

Will define a set of implicit functions

(8.25)

Implicit- function theorem

Given the equation system (8.24) if the fucntions

(8.25)

As in the single – equation case, it is possible to find the partial derivatives of the implicit functions directly from the n equations in (8.24) without having to solve them for the y variables. Taking advantage of the fact that, in the neighborhood N, the equations in (8.24) have the states of identities, we can take the total differential of each of these, and write

8.26)

(8.25) =>

8.27)

We can use (8.27) to eliminate the expressions in (8.26), let us simplify matters by considering only what would happen when alone changes while all the other variables remain constant. ,

(8.27) =>(8.26)=>and dividing by ,

8.28)

(8.28) can be written in matrix notation

By Cramer’s rule

By a suitable adaptation of this procedure, the partial derivatives of the implicit functions with respect to the other variables, can also be obtained.

Example 4

The following three equations

Are satisfied at point P:(x,y,z)=(

If 0 at point P, functions possess continuous derivatives.Ewe can use the implicit function theorem to find the comparative static derivative

We can take the total differentiation of the system

At point P

Therefore, the implicit function rule applies.

Using Cramer’s rule

Example 6

Let the national-income model be written in the form

Endogenous variables (Y,C,T) and take the exogenous variables and parameters ( to be (

(8.30) is a specific case of (8.24) n=3, m=6

positive fractions

Suppose we now hold all the exogenous variables and parameters fixed except then by adapting the result in (8.28)

Government expenditure multiplier

Ex 8.5

1. For each F(x,y)=0 find for each of the following
2. y-6x+7=0
3. 3y+12x+17=0

=3

For each F(x,y)=0 use the implicit function rule to find :

1. F(x,y)=3x2+2xy+4y3=0

=6x+2y

=2x+12y2

1. F(x,y)=12x5-2y=0

1. F(x,y)=7x2+2xy2+9y4=0

1. F(x,y)=6x3 – 3y=0

For each F(x,y,z)=0 use the implicit function rule to find and

1. F(x,y,z)=x3y3+z2+xyz=0
2. F(x,y,z)=x3z2+y3+4xyz=0

Fx=3x2z2+4yz

4. Assuming that the equation F(U, implicitly defines a utility function

a) Find

b) Interpret their respective economic meanings

5. For each of the given equations F(y,x) =0 is an implicit function y=f(x) defined around the point (y=3,x=1)?

1. X3 – 2x2y+3xy2 – 22=0

Fy=– 2x2+3x2y=-

Defined

13=4-2\*33=4-4\*270

Fx=4x+4y/13=4+4\*3=16

Find