Lecture 4

Finding MR Function from AR Function

AR = 15 – Q

TR = AR \* Q = (15 - Q)\*Q = 15Q - $Q^{2}$

MR = $\frac{dTR}{dQ}$ = 15 – 2Q

TR$≡AR\*Q=f\left(Q\right)\*Q$ we can apply the product rule

MR $≡ \frac{dTR}{dQ}=f\left(Q\right)\*1+Q\*f^{'}\left(Q\right)=f\left(Q\right)+Qf^{'}\left(Q\right)$ (1)

$f\left(Q\right)=AR=>MR-AR=MR-f\left(Q\right)=Q\*f'(Q)$ (2) =>

this gives an important relationship between MR and AR: they will always differ by the amount

 $Q\*f'(Q)$; $Q\geq 0$;

$f^{'}\left(Q\right)$ the slope of the AR => AR $\~ P$

$$AR ≡ \frac{TR}{Q}≡\frac{P\*Q}{Q}≡P$$

$AR≡P$ At pure competition:

$$AR≡P=f(Q)$$

 J AR – is horizontal line and

 *f ‘ (Q) =* 0

 MR – AR = 0 = >

 H G MR = AR

 Imperfect competition:

 AR – downward sloping

 K *f ‘ (Q) <* 0 => (2)

O N M Q

MR – AR < 0 for all positive Q, MR curve lie below AR curve

The slope of tangent line $JM=\frac{OJ}{OM}=\frac{HJ}{HG}$

$HG\~Q$ => the distance, by which the MR curve must lie below the AR curve at output N.

$$Nf^{'}\left(N\right)=HG\*\frac{HJ}{HG}=HJ$$

$$KG=HJ$$

Quotient Rule

$$\frac{d}{dx}\*\frac{f\left(x\right)}{g\left(x\right)}=\frac{f^{'}\left(x\right)g\left(x\right)-f(x)g'(x)}{g^{2}(x)}$$

$$\frac{d}{dx}\*\frac{2x-3}{x+1}=\frac{2\left(x+1\right)-\left(2x-3\right)}{\left(x+1\right)^{2}}=\frac{5}{\left(x+1\right)^{2}};$$

$$d \left(\frac{5x}{x^{2}+1}\right)= \frac{5\left(x^{2}+1\right)-5x\*2x}{(x^{2}+1)^{2}}=\frac{5(1-x^{2})}{(x^{2}+1)^{2}};$$

Relationship between MC and AC Functions

As our economic application if the quotient rule, let us consider the rate of change of AC when output varies:

$TC=TC(Q)$ $AC≡\frac{TC(Q)}{Q}$ $Q>0$ the rate of change of AC:

$$\frac{d}{dQ} \frac{TC(Q)}{Q}=\frac{\left[TC^{'}\left(Q\right)\*Q-TC(Q)\right]}{Q^{2}}=\frac{1}{Q}\left[TC^{'}\left(Q\right)+\frac{TC(Q)}{Q}\right]=>for Q>0$$

$$\frac{d}{dQ} \frac{TC(Q)}{Q}\geq \leq 0, if TC^{'}\left(Q\right)\geq \leq \frac{TC\left(Q\right)}{Q} (3)$$

$$=> TC^{'}\left(Q\right)=MC and \frac{TC}{Q}=AC$$

The economic meaning of (3) => the slope of the AC curve will be positive, zero, or negative if and only of the MC curve lies above intersects, or lies below the AC curve

$$TC=Q^{3}-12Q^{2}+60Q$$

$$MC=3Q^{2}-24Q+60$$

$$AC=Q^{2}-12Q+60$$

$$2Q^{2}-12Q=0$$

$$Q^{2}-6Q=0$$

$$Q\left(Q-6\right)=0$$

$$Q=6 Q=0$$

 MC MC

 AC

 AC

 60

 24

 0 6 Q

36 - 12\*6 + 60 = 36 – 12 = 24