

Preparation for Midterm I

1. I. Find the point of intersection of the following two straight lines and draw them if:
- $f_1(x) = -2x - 5$ and $f_2(x) = -10x + 7$;
 - the first line is parallel to the graph of $f(x) = -2x - 5$ and passes through the point $(1; 0)$, while the second line is perpendicular to the graph of $f(x) = -10x + 7$ and has the y-intercept (4) ;
 - the first line is perpendicular to the graph of $f(x) = -2x - 5$ and passes through the point $(1; 0)$, while the second line is parallel to the graph of $f(x) = -10x + 7$ and has the y-intercept (4) ;
 - the first line passes through the points $(1; 0)$ and $(0; 10)$, while the second line passes through the points $(0; 2)$ and $(8; 10)$.
- II. a) Find the point of intersection of two straight lines and draw them if the first line passes through the points $(-1; -5)$ and $(2; 4)$, while the second line is perpendicular to the line $y = -\frac{1}{5}x + 7$ and has y-intercept 6 .
- b) Find the point of intersection of two straight lines and draw them if the first line has the x-intercept 4 and the y-intercept 4 , while the second line is parallel to the line $6x - 3y = 1$ and passes through the point $(1; -4)$.

2. Solve the equations:

a) $\left(\frac{3}{4}\right)^{x-1} \cdot \left(\frac{4}{3}\right)^{\frac{4}{x}} = \frac{9}{16}$;

b) $2^{12x-1} - 4^{6x-1} - 8^{4x-1} = 64$

c) $9^x + 6^x = 2^{2x+1}$;

d) $\log_2 \frac{x^2 - 3x + 3}{4} = -2$

e) $\log_4(2 \log_3(1 + \log_2(1 + 3 \log_3 x))) = \frac{1}{2}$

3. Find the equation of the tangent line to the graph of $f(x)$ at the point $x_0 = 2$. Draw the parabola and tangent line.

a) $f(x) = x^2 - 6x - 7$;

b) $f(x) = 2x^2 + 3x - 20$;

c) $f(x) = (x + 1)^2 - 8$;

d) $f(x) = x^2 - 3x - 8$;

e) $f(x) = x^2 - 6x - 8$;

f) $f(x) = 2(x - 5)^2 - 14$.

4. Calculate the limits using L'Hospital's Rule:

a) $\lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x^2 - 7x + 12}$

b) $\lim_{x \rightarrow 0} \frac{e^{10x} - 1}{\sin 4x}$

c) $\lim_{x \rightarrow 1} \frac{\ln x}{x^2 - 1}$

d) $\lim_{x \rightarrow \infty} \frac{\ln x}{x^2 - 1}$

e) $\lim_{x \rightarrow 1} \frac{(\ln x)^2}{e^{3x-3} - 2x^2 + 1}$

f) $\lim_{x \rightarrow 0} \frac{\log_{11}(1-3x)}{2^x - 2^{-5x}}$

5. Find all asymptotes for the following functions:

a) $f(x) = \frac{5x+1}{3x-2}$

b) $f(x) = \frac{2x^2+3}{x-2}$

c) $f(x) = x^3 / (2(x+1)^2)$

d) $f(x) = 12x - 7 - \frac{2}{3x-3}$

e) $f(x) = \frac{3x^2-1}{x^2-16}$

f) $f(x) = 3x^2 - 2 + 5x$