**Theory of Corporate Finance, ECN 5355**

**Homework for Lecture 5 (9 November)
Please solve the problem in full and submit by email before 5pm on 3 November**

***Problem 1*** *An individual project has pH = 0.9, pL = 0.5, I = 100, R = 125 and B = 10.*

*a) Determine the agency rent, expected pledgeable income and required own wealth of the borrower.*

*5 marks*

Rb ≥ pHB/p 🡺 agency rent = 0.9\*10/0.4 = 22.5

Pledgeable = pH (R – B/Dp)   🡺 Pledgeable = 0.9 (125-10/0.4) = 90

Amin = I – pHR + pHB/p 🡺  Amin = 100-(0.9\*125) +0.9\*(10/0.4) = 10

*b) Assume that two projects, each with the same data as the individual project, can be realized and that returns of the two projects are independent. From the incentive compatibility constraint, determine the agency rent, assuming that the borrower is only rewarded if both projects are successful. Also calculate expected pledgeable income from two projects and required own wealth.*

*5 marks*

Reward of the borrower if successful with both projects:
R2 ≥ 2B/(p (pH + pL)) 🡺 R2 ≥ 35.7

Agency rent: pH22B/(p(pH+pL)) 🡺 0.81\*35.71=28.93 (*2 marks)*

Pledgeable = 2pHR – pH2R2 = 196 *(2 marks)*

2 Amin = 2 I – Pledgeable = 200 – 196 = 4 🡺 Amin ≈ 2 *(1 mark)*

***Problem 2:*** *A single project has the following data:*

*pH = 0.9, pL = 0.4, R = 100, b = 10*

*a) What is the pledgeable income?*

Pledgeable = pH(R – B/p) = 72

*2 marks*

*b) Two borrowers have jointly received a loan. Each of them runs their single project but may monitor the other. Only if both projects are successful does each of them receive the reward Rb. Assume that the cost of monitoring is 15!*

*ba) Assume that the other agent monitors. How much – depending on the unknown Rb - does an agent get if both agents work and she also monitors?*

pH2 Rb – c = 0.81 Rb – 15

*bb) How much does an agent get if she works but fails to monitor (upon which the other agent shirks)?*

pHpL Rb = 0.36 Rb

*bc) How much does an agent get if she neither works nor monitors (assuming the other agent monitors)?*

pL2 Rb + b = 0.16 Rb + 10

*bd) How much does an agent get if she does not work but she monitors (assuming the other agent monitors)?*

pL pH + b – c = 0.36 Rb – 5

*6 marks*

*c) In bb) we assumed that the other agent shirks if the agent does not monitor. But how does monitoring of one agent affect the behavioral incentives of the other agent? How would expressions bc) and bd) have to change if the other agent fails to monitor.*

If the other agent does not monitor, the agent chooses big B which makes shirking attractive.

In bc): pL2 Rb + B = 0.16 Rb + B

In bd):

pL pH Rb + B – c = 0.36 Rb + B – 15

d) Using inequalities ba – bd: how large must Rb be at least to guarantee that both agents want to work and monitor?

0.81 Rb – 15 ≥ max[0.36 Rb,

 0.16 Rb + 10,

 0.36 Rb – 5]

0.81 Rb ≥ max[0.36 Rb +15, (1)

 0.16 Rb + 25, (2)

 0.36 Rb +10] (3)

(3) is implied by (1).

So R15/ = max[15/0.45, 25/0.65) = max[33.3, 38.5] = 38.5

Note that inequality (2) would be violated if instead of b = 10, the agent chooses B>10.