

1. Use the graphing strategy to analyse the function $f(x)$.

Summarize the following information

- a) (max 4 pts) find the domain, intercepts and asymptotes,
- b) (max 4 pts) find the intervals of monotonicity and the local extrema,
- c) (max 4 pts) find the intervals of concavity and the inflection points,
- d) (max 2 pts) and sketch the graph of f .

- I. $f(x) = \frac{x^3 - 4x}{x^2 - 1}$
- II. $f(x) = x^2(x^2 - 1)$
- III. $f(x) = \frac{x}{x^2 - 1}$
- IV. $f(x) = -2 - xe^{-x+2}$

2. (max 6 pts)

I. Find the average value of $f(x) = \sqrt[3]{2-x}$ over the interval $[-5; 0]$.

II. Find the average value of $f(x) = x\sqrt{9-x}$ over the interval $[0; 9]$.

III. Find the average value of $f(x) = \frac{x}{(4-x^2)^2}$ over the interval $[1; 3]$.

3. (max 7 pts)

I. Find the area of the region bounded by $f(x) = 2x^2 - 3x + 9$ and $f(x) = x^2 + 3x + 1$.
Graph the region.

II. Find the area of the region bounded by $f(x) = 2(x-2)^2 - 10$ and $f(x) = -4x + 4$ for $-1 \leq x \leq 4$. Graph the region.

III. Find the area of the region bounded by $f(x) = (x-1)^2 + 1$ and $f(x) = -3x - 2$ for $-1 \leq x \leq 2$. Graph the region.

4. a) (max 4 pts) Solve the system of linear equations by Cramer's rule and Gaussian elimination method:

$$\text{I. } \begin{cases} 3x - 8y - z = -25 \\ 2x - 3y + 2z = -2 \\ -x + 2y - 2z = 1 \end{cases}$$

$$\text{II. } \begin{cases} y + 5x - 3z = -1 \\ z + x + 3y = 3 \\ -y - 2x + 5z = 8 \end{cases}$$

$$\text{III. } \begin{cases} y + 2x = z + 1 \\ 2z - y = 7 - 3x \\ 2y = 8 - x - z \end{cases}$$

$$\text{IV. } \begin{cases} 3P_1 + 2P_2 + P_3 = 28 \\ 2P_1 + 4P_2 - P_3 = 22 \\ 4P_1 + P_2 + 3P_3 = 39 \end{cases}$$

b) (max 4 pts) Solve the matrix equation:

$$\text{I. } X \cdot \begin{pmatrix} 3 & -2 & 4 \\ 7 & 2 & 3 \\ 10 & -1 & 8 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & -1 \\ 1 & 2 & 2 \end{pmatrix}$$

$$\text{II. } \begin{pmatrix} -3 & 1 & 2 \\ 1 & 0 & -1 \\ -4 & 3 & 0 \end{pmatrix} \cdot X = \begin{pmatrix} -2 & 1 & 2 \\ 1 & -1 & 3 \\ 1 & -1 & 4 \end{pmatrix}$$

$$\text{III. } X - X \cdot \begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix}$$

$$\text{IV. } \begin{pmatrix} 1 & -1 \\ -7 & 3 \end{pmatrix} \cdot X + X = \begin{pmatrix} 6 & 1 \\ 2 & -3 \end{pmatrix}$$

5. (max 5 pts)

I. Find the point of intersection of two straight lines and draw them if the first line passes through the points (1 ; -4) and (2 ; 1), while the second line has the slope (-5) and passes through the point (2 ; -3).

II. Find the point of intersection of two straight lines and draw them if the first line is parallel to the line with the equation $2y-3=5x+2$ and passes through the origin, while the second line is perpendicular to the line $3y-2x=4$ and passes through the point (4; 6).

III. Find the point of intersection of two straight lines and draw them if the first line has y-intercept (-5) and x-intercept (-3), while the second line passes through the points (1;-2) and (-5;4).